

## Re: subset of contractible

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- *From:* Zbigniew Karno <[Zbigniew.Karno@xxxxxxxxxx](mailto:Zbigniew.Karno@xxxxxxxxxx)>
  - *Date:* Wed, 11 Jul 2007 03:06:01 -0700
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On 11 Lip, 11:54, Sonya84 <[sonianard...@xxxxxxxxxx](mailto:sonianard...@xxxxxxxxxx)> wrote:

On 11 Lug, 11:41, Sonya84 <[sonianard...@xxxxxxxxxx](mailto:sonianard...@xxxxxxxxxx)> wrote:

Suppose  $X$  is a contractible topological space  
and  $S$  subset of  $X$  is a topological subspace of  $X$ .

Is  $S$  contractible itself?

Since  $X$  is contractible we have that  $X$  is pathwise-connected, so –  
without loss of generality – we can suppose  $X$  is homotopically  
equivalent to a point  $p$  in  $S$ .

Sonia

since  $X$  is contractible to  $p$ , by definition there exists an homotopy  
 $h: X \times [0, 1] \rightarrow X$  such that  $h(*, 0) = \text{id}_X$  while  $h(*, 1) = c_p$   
( $c_p$  denotes the constant which maps every  $x$  in  $X$  to  $p$  in  $X$ ).

So  $h|_S: S \times [0, 1] \rightarrow X$  is such that  $h(*, 0) = \text{id}_S$  while  $h(*, 1) = c_p$ .

The problem is that it ISN'T true that – in general – for every  $t$   
in  $]0, 1[$  and for every  $x$  in  $S$ ,  $h(x, t)$  belongs to  $S$ .

So, why  $S$  need to be contractible if such is  $X$  ?

This means that  $S$  is contractible in  $X$ ,  
however  $S$  is not contractible itself.

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In the case  $S = S^n$ , does not exist a homotopy  
 $F : S \times [0,1] \rightarrow S$  joining  $\text{id}_S$  with a constant  
map.

Regards, Z. Karno

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