

# Re: Graphs and reducibility(or something like that)

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- *From:* "Jon Slaughter" <[Jon\\_Slaughter@xxxxxxxxxxx](mailto:Jon_Slaughter@xxxxxxxxxxx)>
  - *Date:* Sat, 21 Jul 2007 05:36:57 GMT
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"Proginoskes" <[CHeckman@xxxxxxxx](mailto:CHeckman@xxxxxxxx)> wrote in message  
[news:1184985789.069875.300990@xx](mailto:news:1184985789.069875.300990@xx)

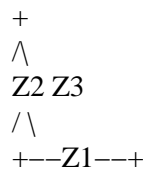
On Jul 20, 6:49 pm, "Jon Slaughter" <[Jon\\_Slaugh...@xxxxxxxxxxx](mailto:Jon_Slaugh...@xxxxxxxxxxx)> wrote:

I'm trying to simplify a network of electrical components and I'm curious if there's any way to do this.

The network is equivalent to a graph but each edge consists of a "weight" to it that is a function. Every edge has a similar function that "looks" the same. Some nodes are known values while others are not.

The function is actually a linear differential equation with constant coefficients.

For example, I might have something like



Where  $Z_1, Z_2, Z_3$  are differential equations.

In reality the graph is equivalent to a system of differential equations and there are unknowns that exist which each branch and/or at some nodes.

I can convert the differential equations into algebraic equations by taking the laplacian as one method but it still involves solving a large system.

I'm more interested in using the fact that each edge/branch is identical to

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every other one except for some basic constants.

In essence I think I can treat each branch with a weight as a vector where the elements of the vector are the coefficients of the DE.

so for example, I might have as a function  $L(t; \dots) = c \cdot dV_1/dt + a \cdot V_2 - b \cdot V_3$

and so I could treat that as a vector like  $(a, -b, c)$ . Every branch will just have different coefficients in the function L but all of the same "form".

I feel that I could somehow use this to my advantage at simplifying the graph/system. (Although I think the size of the vector grows exponentially based on the size of the graph)

Is there anything I can do to simplify my problem? Maybe there are some results in graph theory(I think this is where the problem comes from) that can help? I'm actually wondering if there is some recursive way to simplify the elements. Most of the "vectors" will only have non-zero elements for those that represent coefficients around its branch.

In any case just looking for some ideas,

But where are you using the fact that you have a graph? How does the edge  $uv$  related to the vertex  $u$ , for instance? And what kind of structure do you have? For instance, do the differential equations along a cycle satisfy some law like Kirchoff's?

I don't have a graph?

[http://en.wikipedia.org/wiki/Glossary\\_of\\_graph\\_theory](http://en.wikipedia.org/wiki/Glossary_of_graph_theory)

A graph is just a set of vertices and edges which some relation that maps pairs of vertices to edges? If that's the case then I definitely have a graph.

In any case, when I have all these weights there is an operation to combine them in a specific way.

Suppose

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$W(N_i, N_j)$  is the "weight" of the edge  $N_i N_j$  which, for this problem, is generally some linear differential equation.

Then we know that

$$\sum_{j \in Q} I(W(N_i, N_j)) = 0$$

This is kirchoffs current law which just says that the sum of the currents entering a node is 0.  $I$  is a "function" that maps the weight into its equivalent current and  $Q$  is the set of indecies for nodes that are adjacent to  $N_i$ .

We also would have something like

$$V(N_i) - \sum_{j \in Q} V(N_j) = 0$$

where  $V$  maps a node to a voltage and  $Q$  is an ordered closed list of adjacent nodes. This would be kirchoffs voltage law.

(I'm kinda making up those equations as I go so they might not be totally right)

But I think the main point here is that what I ultimately have is just weights that are differential equations and inside those equations I have all the info about the electronic aspect of the graph but abstractly its just a graph with weights and I should be able to simplify it.

Whats really going on is that I have the weight function  $W(N_i, N_j) = (V(t, N_i) - V(t, N_j)) / Z_{N_i N_j}(t, N_1, \dots, N_n; a_1 \dots a_m)$

And you can think of this as ohms law. It essentially defines a current on the edge  $N_i N_j$ . Then of course I must have  $\sum W(N_i, N_j) = 0$  for all  $N_j$  with common vertex  $N_i$ . This is kirchoffs current law.

Maybe I'm starting not to make sense though. The idea is simply to generalize the case for resistors.

if I have some graph of resistors then what I really have is a linear algebraic system of equation. Each edge creates an equation. The system is related to the graph. Change the graph and change the system.

Infact what we have is  $V(N_i) - V(N_j) - R_{N_i N_j} I_{N_i N_j} = 0$  for each edge and  $\sum I_{N_i N_j} = 0$  for each node  $N_i$  where the sum is over adjacent nodes to  $N_i$ .

It seems I end up having to have an extra structure though that exists on the graph. Maybe I need to think it out a little more.

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Thanks,  
Jon

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