Re: cube root of a given number

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- *Date*: Sun, 22 Jul 2007 04:13:23 −0000

Hey you, sttscitrans@xxxxxxxxx,

Did you dare to mention SQUARE ROOTS when talking about your alleged GENERAL HURTWITZ's ROOT−SOLVING METHOD? (pun intended) Very funny, indeed.

please , do a favor to the sci.math audience, ask to HURTWIZT to tell you if by means of FAREY FRACTIONS he pubished something similar to all what follows. I am sure you have the cheek to say that you have read all what follows in many books on numbers, even in ancient clay tablets. (Pun intended)
Of Course this challenge apply also to your friend.
Wait a minute, please do not tell me that your math teacher and even HURTWITZ never told you anything about what follows. What a shame, indeed, worst when considering that I am only bringing to you a TRIVIAL EXAMPLE ON SQUARE ROOTS. What a shame, indeed.

Notice, that what follows is neither about Farey Fractions, nor Continued Fractions, nor Pell's equation, nor pigeon holes, nor birds, nor cows, but just about a simple example on the GENERAL ROOT SOLVING METHODS shown in my book and webpages, such methods have no precedents in the whole history of mathematics and certainly yields best approximations. Could you ever understand or accept that? i do not think so, because this hurts so much, indeed, and I understand you, and i DON'T care, and I will continue spreading this CRUDE TRUTH, and you will not be able to prevent people from reading all this.

All this comments comes from the contents of the webpage:
http://mipagina.cantv.net/arithmetic/rmdef.htm
and the book: LA QUINTA OPERACIÓN ARITMÉTICA. Arithmonic Mean.
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The Rational Mean of the fractions: f1=a1/b1 and f2=a2/b2 is:

Rm[f1, f2] = (a1+a2)/(b1+b2)

By agency of such a simple arithmetical operation you can produce all the Householder expressions for the Nth root of any number P.
Notice that if you change the form of the fraction a1/b1= (x/x)*(a1/b1)= (x*a1)/(x*b1) then you will get another result (provided that x is not equal to 1):

For example: Rm[(x/x)*f1, f2]= (x*a1 + a2) / (x*b1 + b2)

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Higher−order rational process based on the Rational Mean:

FUNDAMENTAL PRINCIPLE:
Any two fractions whose product is P represent two rational approximations −by defect and excess−− to the square root of P.

If departing from those two fractions you can compute two mean values whose product is also P then you have another two closer approximations to the root. By continuing this process you will get a root−solving algorithm for the square root of P, moreover, by using the Rational Mean you will get a higher−order root−solving algorithm.

Starting with a set of two fractions f1, f2 whose product is f1*f2 = P.
For example:
f1 =x/1 f2=P/x

Compute the following two rational means:

Rm[(x/x)*f1, f2] = (P+x^2) / (2x) (Newton)

Rm [(P/P)*f1, (x/x)*f2]= (2Px) / (P+x^2)

It yields, two expressions whose product is trivial and equal to P and are closer to the square root of P.
You can use each of those new functions as independent iterating functions both of them converging quadratically.

If you don't like quadratic convergence then compute another two similar rational means by previously assigning those new functions to f1 and f2, as follows:

f1 = (P+x^2) / (2x)
f2 = (2Px) / (P+x^2)

The two new rational means yields:
Rm [(x/x)*f1, f2] = (x^3 + 3Px) / (P

+3x^2) (Halley)
Rm [(P/P)*f1, (x/x)*f2]= (P^2 + 3Px^2) / (x^3 + 3Px)

two expressions whose product is trivial and equal to P, both of them multiply by THREE the number of exact digits in each iteration.

If you prefer more convergence speed, then make:

f1 = (x^3 + 3Px) / (P+3x^2)

f2 = (P^2 + 3Px^2) / (x^3 + 3Px)

and compute other two rational means:

Rm [(x/x)*f1, f2] = (x^4 + 6Px^2 + P^2) / (4x^3 + 4Px)
(Householder)
Rm [(P/P)*f1, (x/x)*f2]= (4Px^3 + 4xP^2)/ (x^3 + 3Px)

Two expressions whose product is trivial and equal to P, both of them multiply by FOUR the number of exact digits in each iteration.

By continuing this process, in the next step you will get two functions which multiply by five the number of exact digits in each iteration.
And so on...
That is, you will get all the Householder expressions for the square root along with another iterating function.

Believe it or Not!.
Based on the evidence at hand, this so naïve, trivial, natural and simple rational process has no precedents since Sumerians times up to now. We have not used neither any Cartesian−decimal system, nor any derivatives, nor infinitesimal calculus, at all
I think, many experts on the history of mathematics should cogitate on the very long story on root−solving. Indeed, it is disturbing to realize these so simple rational processes based on the rational mean do not appear in any book on numbers since ancient times up to now.

All this is fully explained in my book:
LA QUINTA OPERACIÓN ARITMÉTICA. Arithmonic Mean © Domingo Gomez Morin.

and its webpage:
http://mipagina.cantv.net/arithmetic

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