Re: Graphs and reducibility(or something like that)

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- *From*: "Jon Slaughter" <Jon_Slaughter@xxxxxxxxxxx>
- *Date*: Sun, 22 Jul 2007 01:14:12 −0500

"Proginoskes" <CCHeckman@xxxxxxxxx> wrote in message
news:1185082922.031088.289140@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

> On Jul 21, 12:36 am, "Jon Slaughter" <Jon_Slaugh...@xxxxxxxxxxx>
> wrote:
>
> > "Proginoskes" <CCHeck...@xxxxxxxxx> wrote in message
> > news:1184985789.069875.300990@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
> >
> > > On Jul 20, 6:49 pm, "Jon Slaughter"
> > > <Jon_Slaugh...@xxxxxxxxxxx> wrote:
> > >
> > > > I'm trying to simplify a network of electrical
> > > > components and I'm
> > > > curious if
> > > > theres any way to do this.
> > > >
> > > > The network is equivalent to a graph but
> > > > each edge consists has a
> > > > "weight"
> > > > to it that is a function. Every edge has a
> > > > similar function that
> > > > "looks" the
> > > > same. Some nodes are known values while
> > > > others are not.
> > > >
> > > > The function is actually a linear differential
> > > > equation with constant
> > > > coefficients.
> > > >
> > > > For example, I might have something like

```
+
/\
Z2 Z3
/ \
+−−Z1−−+
```

Where Z1, Z2, Z3 are differential equations.

In reality the graph is equivalent to a system of differential
equations and
there are unknowns that exist which each branch and/or at some nodes.

I can convert the differential equations into algebraic equations by
taking
the laplacian as one method but it still involves solving a large
system.

I'm more interested in using the fact that each edge/branch is
identical to
every other one except for some basic constants.

In essence I think I can treat each branch with a weight as a vector
where
the elements of the vector are the coefficients of the DE.

so for example, I might have as a function L(t;....) = c*dV1/dt +
a*V2 − b*V3

and so I could treat that as a vector like
(a,−b,c). Every branch will
just
have different coefficients in the function L
but all of the same
"form". I
feel that I could somehow use this to my
advantage at simplifying the
graph/system. (Although I think the size of
the vector grows
exponentially
based on the size of the graph)

Is there anything I can do to simplify my
problem? Maybe there are
some
results in graph theory(I think this is where
the problem comes from)
that
can help? I'm actually wondering if there is
some recursive way to
simplify
the elements. Most of the "vectors" will only
have non−zero elements
for
those that represent coefficients around its
branch.

In any case just looking for some ideas,

But where are you using the fact that you have a graph? How
does the
edge uv related to the vertex u, for instance? And what kind
of
structure do you have? For instance, do the differential
equations
along a cycle satisfy some law like Kirchoff's?

I don't have a graph?

http://en.wikipedia.org/wiki/Glossary_of_graph_theory

> A graph is just a set of verticies and edges which some relation that
> maps
> pairs of verticies to edges? If thats the case then I definitely have a
> graph.

> Oh, I see now. You are given an electrical network, which gives you
> the graph.

Yes.

> In any case, when I have all these weights there is an operation to
> combine
> them in a specific way.
>
> Suppose
>
> W(Ni,Nj) is the "weight" of the edge NiNj which, for this problem, is
> generally some linear differential equation.
>
> Then we know that
>
> sum(I(W(Ni,Nj)),j \in Q) = 0
>
> This is kirchoffs current law which just says that the sum of the
> currents
> entering a node is 0.

> There is another law which states that the sum of the current around a
> loop must add up to 0.

Well, the voltage drops...

> I is a "function" that maps the weight into its
> equivilent current and Q is the set of indecies for nodes that are
> adjacient
> to Ni.
>
> We also would have something like
>
> V(Ni) − sum(V(Nj),j \in Q) = 0
>
> where V maps a node to a voltage and Q is an ordered closed list of

adjacient nodes. This would be kirchoffs voltage low.

(I'm kinda making up those equations as I go so they might not be totally right)

I think I can see at least the general direction you're heading.

Whatever the laws are, they give you a system of equations. Kirchoff's two laws will state that these equations are related to each other, and so your system will be redundant. The redundancies, along the edges, will be determined by the cycle−space of your graph G (a cycle is basically a loop; you don't repeat vertices or edges until you're done with walking around a cycle).

Right. And theres a lot of redundancy. Although this happens with or without the branches all being "functionally" the same.

You can find the cycle space by setting up a particular matrix and looking for a basis for its null space.

That might work. Although I have to figure out how to generate the matrix easily. I guess each entry would represent an edge? (so each entry in the matrix could represent the current, which is actually represented as a solution to a differential equation)

But I think the main point here is that what I ultimately have is just weights that are differential equations and inside those equations I have all the info about the electronic aspect of the graph but abstractly its just a graph with weights and I should be able to simplify it.

Whats really going on is that I have the weight function W(Ni,Nj) = (V(t,Ni) − V(t,Nj))/Z_NiNj(t,N1,...,Nn;a1..am)

And you can think of this as ohms law. It essentially defines a current on
the edge NiNj. Then of course I must have sum(W(Ni,Nj)) = 0 for all Nj with
common vertext Ni. This is kirchoffs current law.

Maybe I'm starting not to make sense though. The idea is simply to generalize the case for resistors.

Basically: "If you put other gadgets into the circuit (capacitors,

> relays, whatever), you have other rules which determine how the
> current flows through them", which are given by the differential
> equations? And you're trying to simplify your system.

Yes.

I think essentially if A_ij is the ijth element of a matrix, then A_ij is the current along the branch bound by the ith and jth nodes.

But the currents are solutions to differential equations involving other currents and the "functional form" of the branch itself.

> > if I have some graph of resistors then what I really have is a linear
> > algebraic system of equation. Each edge creates an equation. The system
> > is
> > related to the graph. Change the graph and change the system.
> >
> > Infact what we have is V(Ni) − V(Nj) − RNiNj*INiNj = 0 for each edge and
> > sum(NiNj) = 0 for each node Ni where the sum is over adjacient nodes to
> > Ni.
> >
> > It seems I end up having to have an extra structure though that exists on
> > the graph. Maybe I need to think it out a little more.
>
> Well, hopefully I've thrown out some useful ideas here, or failing
> that, the sort of questions to ask. 8−)

Yes, it helps a little(just because it gets me thinking about it). I think maybe I can mess around with the matrix idea and see what happens. I think the problem is going to be that the elements of the matrix are not real or complex numbers but vectors or differential equations. This is going to make it harder to deal with but maybe the process is easier.

If, say, I have

```
V2
+
/\
Z2 Z3
/ \
V1 +−−Z1−−+ V3
```

and the the current I_i is the current through the branch B_i which is equivilent to Z_i

The V_i's are voltages but also alias them as the nodes N_i.

So the "current matrix" would be 3x3 and look like

0 (V2−V1)/Z2 (V3−V1)/Z1
(V1−V2)/Z2 0 (V3−V2)/Z3
(V1−V3)/Z1 (V2−V3)/Z3 0

As you can see, in this case its symmetric which helps. That is not a property of the "functional form" of the Z's though.

Also each element is really an expression involving unknowns. In this case I would have to specify two of the 3 V's to get a solution.

In this case, sum(row) = 0 because of conservation of charge. I believe that should hold in general.

(because row i is the ith node and sum of currents going into that node(which are given by the columns) must be 0)

I'll need to think about it some more but it does seem like it actually might work out.

Any more ideas? ;)

Thanks,
Jon

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