

Re: Graphs and reducibility(or something like that)

# Re: Graphs and reducibility(or something like that)

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- *From:* "Jon Slaughter" <[Jon\\_Slaughter@xxxxxxxxxxx](mailto:Jon_Slaughter@xxxxxxxxxxx)>
  - *Date:* Sun, 22 Jul 2007 01:14:12 -0500
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"Proginoskes" <[CCHeckman@xxxxxxxxxxx](mailto:CCHeckman@xxxxxxxxxxx)> wrote in message  
<news:1185082922.031088.289140@xx>

On Jul 21, 12:36 am, "Jon Slaughter" <[Jon\\_Slaugh...@xxxxxxxxxxx](mailto:Jon_Slaugh...@xxxxxxxxxxx)>  
wrote:

"Proginoskes" <[CCHeck...@xxxxxxxxxxx](mailto:CCHeck...@xxxxxxxxxxx)> wrote in message  
<news:1184985789.069875.300990@xx>

On Jul 20, 6:49 pm, "Jon Slaughter"  
<[Jon\\_Slaugh...@xxxxxxxxxxx](mailto:Jon_Slaugh...@xxxxxxxxxxx)> wrote:

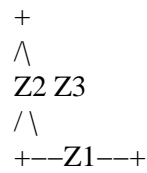
I'm trying to simplify a network of electrical  
components and I'm  
curious if  
theres any way to do this.

The network is equivalent to a graph but  
each edge consists has a  
"weight"  
to it that is a function. Every edge has a  
similar function that  
"looks" the  
same. Some nodes are known values while  
others are not.

The function is actually a linear differential  
equation with constant  
coefficients.

For example, I might have something like

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Where  $Z_1, Z_2, Z_3$  are differential equations.

In reality the graph is equivalent to a system of differential equations and there are unknowns that exist which each branch and/or at some nodes.

I can convert the differential equations into algebraic equations by taking the laplacian as one method but it still involves solving a large system.

I'm more interested in using the fact that each edge/branch is identical to every other one except for some basic constants.

In essence I think I can treat each branch with a weight as a vector where the elements of the vector are the coefficients of the DE.

so for example, I might have as a function  $L(t; \dots) = c \cdot dV_1/dt + a \cdot V_2 - b \cdot V_3$

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and so I could treat that as a vector like  
(a,-b,c). Every branch will  
just  
have different coefficients in the function L  
but all of the same  
"form". I  
feel that I could somehow use this to my  
advantage at simplifying the  
graph/system. (Although I think the size of  
the vector grows  
exponentially  
based on the size of the graph)

Is there anything I can do to simplify my  
problem? Maybe there are  
some  
results in graph theory(I think this is where  
the problem comes from)  
that  
can help? I'm actually wondering if there is  
some recursive way to  
simplify  
the elements. Most of the "vectors" will only  
have non-zero elements  
for  
those that represent coefficients around its  
branch.

In any case just looking for some ideas,

But where are you using the fact that you have a graph? How  
does the  
edge uv related to the vertex u, for instance? And what kind  
of  
structure do you have? For instance, do the differential  
equations  
along a cycle satisfy some law like Kirchoff's?

I don't have a graph?

[http://en.wikipedia.org/wiki/Glossary\\_of\\_graph\\_theory](http://en.wikipedia.org/wiki/Glossary_of_graph_theory)

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A graph is just a set of vertices and edges which some relation that maps pairs of vertices to edges? If that's the case then I definitely have a graph.

Oh, I see now. You are given an electrical network, which gives you the graph.

Yes.

In any case, when I have all these weights there is an operation to combine them in a specific way.

Suppose

$W(N_i, N_j)$  is the "weight" of the edge  $N_i N_j$  which, for this problem, is generally some linear differential equation.

Then we know that

$$\sum_{j \in Q} I(W(N_i, N_j)) = 0$$

This is Kirchhoff's current law which just says that the sum of the currents entering a node is 0.

There is another law which states that the sum of the current around a loop must add up to 0.

Well, the voltage drops...

$I$  is a "function" that maps the weight into its equivalent current and  $Q$  is the set of indices for nodes that are adjacent to  $N_i$ .

We also would have something like

$$V(N_i) - \sum_{j \in Q} V(N_j) = 0$$

where  $V$  maps a node to a voltage and  $Q$  is an ordered closed list of

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adjacent nodes. This would be kirchoffs voltage law.

(I'm kinda making up those equations as I go so they might not be totally right)

I think I can see at least the general direction you're heading.

Whatever the laws are, they give you a system of equations. Kirchoff's two laws will state that these equations are related to each other, and so your system will be redundant. The redundancies, along the edges, will be determined by the cycle-space of your graph  $G$  (a cycle is basically a loop; you don't repeat vertices or edges until you're done with walking around a cycle).

Right. And there's a lot of redundancy. Although this happens with or without the branches all being "functionally" the same.

You can find the cycle space by setting up a particular matrix and looking for a basis for its null space.

That might work. Although I have to figure out how to generate the matrix easily. I guess each entry would represent an edge? (so each entry in the matrix could represent the current, which is actually represented as a solution to a differential equation)

But I think the main point here is that what I ultimately have is just weights that are differential equations and inside those equations I have all the info about the electronic aspect of the graph but abstractly it's just a graph with weights and I should be able to simplify it.

What's really going on is that I have the weight function  $W(N_i, N_j) = (V(t, N_i) - V(t, N_j)) / Z_{N_i N_j}(t, N_1, \dots, N_n; a_1 \dots a_m)$

And you can think of this as ohm's law. It essentially defines a current on the edge  $N_i N_j$ . Then of course I must have  $\sum W(N_i, N_j) = 0$  for all  $N_j$  with common vertex  $N_i$ . This is kirchoff's current law.

Maybe I'm starting not to make sense though. The idea is simply to generalize the case for resistors.

Basically: "If you put other gadgets into the circuit (capacitors,

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relays, whatever), you have other rules which determine how the current flows through them", which are given by the differential equations? And you're trying to simplify your system.

Yes.

I think essentially if  $A_{ij}$  is the  $ij$ th element of a matrix, then  $A_{ij}$  is the current along the branch bound by the  $i$ th and  $j$ th nodes.

But the currents are solutions to differential equations involving other currents and the "functional form" of the branch itself.

if I have some graph of resistors then what I really have is a linear algebraic system of equation. Each edge creates an equation. The system is related to the graph. Change the graph and change the system.

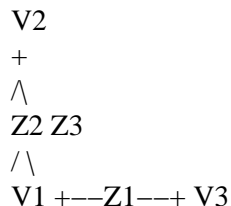
Infact what we have is  $V(N_i) - V(N_j) - R_{NiNj} * I_{NiNj} = 0$  for each edge and  $\sum(N_iN_j) = 0$  for each node  $N_i$  where the sum is over adjacent nodes to  $N_i$ .

It seems I end up having to have an extra structure though that exists on the graph. Maybe I need to think it out a little more.

Well, hopefully I've thrown out some useful ideas here, or failing that, the sort of questions to ask. 8-)

Yes, it helps a little(just because it gets me thinking about it). I think maybe I can mess around with the matrix idea and see what happens. I think the problem is going to be that the elements of the matrix are not real or complex numbers but vectors or differential equations. This is going to make it harder to deal with but maybe the process is easier.

If, say, I have



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and the the current  $I_i$  is the current through the branch  $B_i$  which is equivalent to  $Z_i$

The  $V_i$ 's are voltages but also alias them as the nodes  $N_i$ .

So the "current matrix" would be 3x3 and look like

$$\begin{matrix} 0 & (V_2-V_1)/Z_2 & (V_3-V_1)/Z_1 \\ (V_1-V_2)/Z_2 & 0 & (V_3-V_2)/Z_3 \\ (V_1-V_3)/Z_1 & (V_2-V_3)/Z_3 & 0 \end{matrix}$$

As you can see, in this case its symmetric which helps. That is not a property of the "functional form" of the  $Z$ 's though.

Also each element is really an expression involving unknowns. In this case I would have to specify two of the 3  $V$ 's to get a solution.

In this case,  $\text{sum}(\text{row}) = 0$  because of conservation of charge. I believe that should hold in general.

(because row  $i$  is the  $i$ th node and sum of currents going into that node(which are given by the columns) must be 0)

I'll need to think about it some more but it does seem like it actually might work out.

Any more ideas? ;)

Thanks,  
Jon

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