

Re: Uncountable sets

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- *From:* Proginoskes <CHeckman@xxxxxxxx>
 - *Date:* Mon, 23 Jul 2007 02:34:15 -0000
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On Jul 22, 9:22 pm, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxxx> wrote:

On Jul 22, 3:15 pm, The World Wide Wade <aderamey.a...@xxxxxxxx>
wrote:

In article <1185079000.688698.217...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>,

Proginoskes <CHeck...@xxxxxxxx> wrote:

On Jul 21, 10:18 pm, S J Kissane <skiss...@xxxxxxxx>
wrote:

Hi

My intuition says that uncountable sets do not exist. I have no problem with the existence of aleph-null; I just have a problem with the existence of anything bigger. Is there a name for this intuition?

Yes, around here it's called "being an anti-Cantor crank".

Many "anti-Cantor" mathematicians – and certainly they are not all cranks – allow for the existence of uncountable sets.

I think you're not referring to the notion that Cantor's naive set theory (with a universe), the Mengenlehre, is not ZF(C), the Zermelo–Fraenkel set-theoretic (by default in definition) theory of sets,

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where the axioms of ZF variously amended with choice/well-ordering (the C) describe a collection of all things that might be sets.

A primary difference between naive set theory, which might by itself include fanciful notions of sets containing themselves (non-well-founded, irregular sets), and the set theory of Zermelo is the axiomatization, or fiat, that there are not irregular sets, via the axiom of regularity, also known as the axiom of well-foundedness. Where each of the other axioms of ZF set theory was enlisted to enrich or broaden the set-theoretic universe (domain of discourse), in expanding comprehension, the axiom of regularity instead is to restrict comprehension. The regularity of sets was axiomatized, in a sense made fiat or demanded, because otherwise Russell's specification of a set, of one of the paradoxes named after Bertrand Russell in set theory, would enable that given the existence of a universe of sets, there would be a set that while specified to contain only sets that don't contain themselves, would thus contain itself, the Russell paradox, where there is also the Russell-Myhill paradox to consider, which is another case showing that universal quantification, of a sort, over a comprehensive (or comprehended) universe of objects, would lead to that variously sets of a given largeness were irregular, that infinite sets bijected to infinite powerset, and so on.

Where it might be so that those antinomies are seen to afflict basically any theory rich enough to describe itself, instead perhaps the emphasis should be upon the notion that those results illustrate something deeper about the fundamental structures themselves: they do not fit the ideal naive intuition as has seen place since (and in many ways since long before) the specification of Peano's axioms, for example, but that instead those INFINITE collections have various properties that are quite dissimilar to those of finite ones, for example, that the set of natural integers contains itself, in compactification of the set towards completion of the numbers, and in particular resolution of Zeno's paradoxes.

ZF, Zermelo-Fraenkel set theory, partially eponymous for Fraenkel who called transfinite cardinals a disease from which mathematical foundations would one day recover, has as its domain of discourse some few elements of a less restricted universe. (So, add Fraenkel to the list of those disagreeing with the conclusions of transfinite cardinals, not "anti-Cantor cranks".) Then, where the specification of those elements is exactly the same as the specification of the Russell set of the soi-disant "paradox" of Russell, the collection is seen to contain itself.

So, ZF is inconsistent. Quantify over sets or don't.

Ross

Congratulations to The World Wide Wade for taking my comment out of

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context. Right after

Yes, around here it's called "being an anti-Cantor crank".

I wrote:

But seriously ...

indicating that I was joking, but this was clipped.

--- Christopher Heckman

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