Re: Is it permitted in math to go beyond?

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- *From*: Hero <Hero.van.Jindelt@xxxxxx>
- *Date*: Wed, 25 Jul 2007 11:24:04 −0700

Carlie wrote:

> Hero wrote:
>
>> ... Nikos Drakos wrote:
>> "... in the hyperbolic plane three non−collinear points lie either on a circle, a horocycle, or a hypercycle accordingly, as the perpendicular bisectors of the triangle are concurrent in an ordinary point, an ideal point, or an ultraideal point."
>> http://www.math.uncc.edu/~droyster/math3181/notes/hyprgeom/node68.html
>>
>> This seems strange and familiar at the same time to me. Is this sentence really true?
>
> Yes. It's a standard result in hyperbolic plane geometry.

And in a second letter Charlie wrote:

> The Poincare disk might be the best way of thinking about these points. The Poincare disk is the set of all points in the interior of the unit disk: x^2 + y^2 < 1. Now think about all the circles orthogonal to the unit circle (orthogonal: they intersect the unit circle and the tangents at the points of intersection are perpendicular). The portion of those circles within the Poincare disk are the lines −− often called h−lines. One can show that the Poincare disk with the h−lines is a model of the hyperbolic plane.
>
> Now this is all embedded in the Euclidean plane, and one can take advantage of this embedding in order to deduce a lot of things about the hyperbolic plane −− for example, much of hyperbolic trigonometry is developed this

> way. So, one way to think of the ideal and ultra−ideal points is to visualize this embedding. The ideal points lie on the unit circle x^2 + y^2 = 1, while the ultra−ideal points satisfy that x^2 + y^2 > 1.
>
> One would not routinely use a hyperbolic metric on these points, just as one would not use a Euclidean metric on ideal points in the real projective plane. They would be infinitely far away −− in the hyperbolic measure.

My trouble here is: When three points are lying on a hypercycle then the perpendicular bisectors of the triangle are h−straight−lines, which do not meet inside and on the edge of the disc. Prolonged they never meet − these h−straight−lines are depicted in the poincare−disc as arcs of circles, which do not intersect also when prolonged beyond the edge of the disc. This is contrary to what was stated from Nikos, isn't it?

Or the same expressed differently: In the Klein−disc two straight lines are hyper−parallel, when these lines prolonged through the edge of the disc interect outside in an ultra−ideal point.

From Klein to Poincare i have to do parallel projection onto half of a

sphere, followed by stereographic projection onto a plane. Already with the first projection i'm stuck: whereto these ultra−ideal points move?

With friendly greetings
Hero

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