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- *From:* Robert Israel <israel@xx>
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Pawel Gladki <gladki@xxxxxxxxxxxxxxxxxxxxxxxx> writes:

Hello,

alainverghote@xxxxxxxx wrote:

Are there known ways to solve
 $g(x+1)=g(x)+x+1/x$, x in \mathbb{R}^+ ,
 g a continuous function $g:\mathbb{R}^+ \rightarrow \mathbb{R}^+$

Yes, there are :) This is an equation of the type:

$$g(x+a) - bg(x) = f(x)$$

and the general solution is of the form:

$$g(x) = T(x) * b^{\{x/a\}} + g_0(x),$$

where $T(x) = T(x + a)$ is an arbitrary periodic function of period a , and g_0 is any particular solution of the nonhomogeneous equation. For more details see:

<http://eqworld.ipmnet.ru/en/solutions/fe/fe1108.pdf>

and the references cited there.

... and a particular solution in this case is $g(x) = (x^2-x)/2 + \Psi(x)$,
where $\Psi(x) = (d/dx) \ln(\Gamma(x))$.

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