

Re: how to list all of the real numbers

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- *From:* "Ross A. Finlayson" <raf@xxxxxxxxxxxxxxxx>
 - *Date:* Sat, 11 Aug 2007 17:37:51 -0700
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On Aug 9, 4:40 pm, lwal...@xxxxxxxx wrote:

On Aug 9, 9:24 am, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxxx> wrote:

Denying the Axiom of Infinity doesn't necessarily make on a finitist: instead perhaps there is some consideration that the characterization of an infinite set by the axiom of infinity as generally stated (that

I see what you mean. In other words, what you are suggesting is that one replace the Axiom of Infinity with another axiom ϕ , such that ZF-Infinity+ ϕ proves the existence of a set that is not hereditarily finite, but one can prove certain theorems that match one's intuition yet whose negations are provable in ZFC.

No, not everyone who thinks Cantor is not the end—all be—all when it comes to infinity is a crank.

But standard mathematicians, of course, believe otherwise.

If a function bijects between the natural integers and reals then it shares a variety of properties with the Equivalency Function, which maps the first integer after zero to the first real number after zero in the total linear and well-ordering there of the reals, the natural order. That the value EF(1) would be not only an infinitesimal, hinting that the real numbers contain infinitesimals, but furthermore a value nearer zero than any other value, contradicting the density of the reals within themselves as always having infinitely many between any two, leads to some non-standard notions of the real numbers. I wouldn't characterize the notion of rejection of the Axiom of Infinity that way, because I think a true specification or "axiomatization" of infinity would see there a contradiction with the Axiom of Regularity, if for example infinite sets were irregular.

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So, for example, instead of "ZF – AoI + phi", for some alternate formulation phi of the specification of the existence of an infinite set, it would be more along the lines of "ZF – AoI – AoR + phi", for some phi expressing that alternate specification of the existence of an infinite set and perhaps anti–foundation.

With regards to the transfer principle, you might consider Schmieden and Laugwitz formulation (or rather, model) of the real numbers where besides being the complete ordered field, in terms of various operations upon real numbers, that the set that comprises the continuum of the real numbers has as well structure as a "partially ordered ring with rather restricted transfer principle", as I believe was coined to express their conception of the real numbers. That is to say, as I have described here in some detail, there is a consideration that given various operations on the real numbers, they form the complete ordered field, but as well there is another set of operations, basically able to find successor reals or viewing real numbers as a contiguous sequence of points.

(The antidiagonal argument doesn't work in unary, binary, trinary, or "base infinity". The unit impulse function is not a real function.)

With regards to the notions of infinitesimals and where they exist the notion that there are infinitesimals somewhere among the reals, the reals being gapless thus there being infinitesimals in the reals, thus that some real numbers have properties that do not meet the standard specifications of the reals numbers, that leads to a consideration that the Dedekind/Cauchy method of expressing real numbers, where Dedekind's is basically equivalent to I believe Eudoxus' construction some 2000 years previous and equivalent to Cauchy's, that D/C cuts/sequences are not adequate to represent the real numbers. There are only real numbers between zero and one, if there are infinitesimals between zero and one they're real numbers, elements of the continuum.

(There exist counterexamples in standard real analysis showing a least positive real.)

I think most mathematicians don't bother creating, or assimilating, an entire theory of mathematics, with the intuitionistic notions of given truisms that are plenty upon which to build useful, pragmatic results. It's not necessary for them that they do, besides, much of foundations is specialist and technical, expressly and tediously verbose in formalization.

Ross

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I agree that infinitesimals are possible, but unfortunately most mathematicians will only consider standard theories.