Re: how to list all of the real numbers

Source: http://sci.tech−archive.net/Archive/sci.math/2007−08/msg02055.html

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- *From*: "Ross A. Finlayson" <raf@xxxxxxxxxxxxxxx>
- *Date*: Mon, 13 Aug 2007 12:32:55 −0700

On Aug 11, 10:47 pm, lwal...@xxxxxxxxx wrote:

> On Aug 11, 7:29 pm, Virgil <vir...@xxxxxxxxxxx> wrote:
>
> > Most mathematicians will consider systems of infinitesimals possible,
> > but unless their own work makes such infinitesimals useful, they are not
> > liable to spend much energy or time on them.
>
> "Crank" #1: Why accept the Axiom of Infinity?
> Standard Mathematician: Because it is useful.
> "Crank" #2: Why reject infinitesimals?
> Standard Mathematician: Because they are not useful.
>
> So far in both this thread and in "An Inconvenient Truth" the
> consensus
> among standard mathematicians is that the infinite sets are "useful,"
> (in response to HdB's claims that the Axiom of Infinity is not useful)
> while infinitesimals are not "useful," which is why the former is part
> of standard (or classical) analysis while the latter is relegated to
> Nonstandard Analysis.
>
> The difference? The best argument I've seen is that it's not the
> infinite sets themselves that are "useful," but the complete ordered
> field that is important to the solution of differential equations. It
> was stated that a complete ordered field cannot contain
> infinitesimals,
> for the set of infinitesimals are a set with no least upper bound. In
> NSA differential equations become difference equations (with
> infinitesimal differences), while integrals become sums of infinitely
> many infinitesimals. And since differential equations and integrals
> are easier to work with than (even finite) difference equations and
> summations, infinitesimals are to be rejected. But one must assume
> the Axiom of Infinity in order to prove that a complete ordered field
> even exists.

Well−order the reals, i.e., biject the set of real numbers to an initial segment of an ordinal. Then, apply "Cantor's first" or nested intervals as it is sometimes called, that each sucessive element brackets the remaining interval. Then, besides whether c = Aleph_1, Aleph_2, ..., where it is said to be consistently equal to each of those with the undecideable continuum hypothesis, yet none of them, besides that, consider: does there not always exist each real number of a segment of the continuum in the remaining bracket/interval?

That is where, if some initial segment of the ordinal O equivalent to c led to a degenerate interval, for example [0,0] containing only the point/scalar/number 0, then there is a question as to whether the continuum's cardinality is thus simply equivalent to a lesser cardinal than O's. Similarly to how other considerations lead to that the real numbers have a cardinality greater than any given ordinal, it leads to an argument that they then as well have a cardinality less than any given ordinal to which they are equivalent. This was discussed further some year and so ago in "On well−ordering(s) of the reals, infinity."

Browsing Hardy's "A course of pure mathematics", I consider this:

The 'real number' x, with which we have been concerned in the two preceding chapters, may be regarded from many different points of view. It may be regarded as a pure number, destitute of geometrical significance, or a geometrical significance may be attached to it in at least three different ways. It may be regarded as _the measure of a length_, viz. the length $A_0 P$ along the line Λ of Ch. I. It may be regarded as _the mark of a point_, viz. the point P whose distance from A_0 is x. Or, it may be regarded as _the measure of a displacement_ or _change of position_ on the line Λ.

(Here, underscores delimit the author's emphasis and dollar signs indicate TeX linear math mode, and "viz." is an abbreviation of "videlicet.")

Then, where a real may be a mark of a point, consider whether Hardy would accept that each mark of a point, or simply point, would represent a real number. Then, there is a question: how can there be marked each point on the unit interval? Cantor's nested intervals rreesult would have that any attempt to stipple or pock the line into existence would fail. Yet, drawing from mark to mark has that any point so marked is only as a result of a generative process that each new point in the course of the line is immediately adjacent to: the previous point.

(Consider as an aside: There is a notion that where the real numbers exist, that synthetically there exists a, say, canonical infinite−dimensional orthogonal vector basis. Then, consider a unit length segment from the origin, a vector. Where it demands two non−zero components, each is less than 1, three, each further less, until for the unit length vector to be uniquely and only expressible in terms of components that span the space, each is, or almost all are, infinitesimal.)

It was mentioned that the reason that rigor/soundness in analysis is a perceived requirement is for the differentialists, or analysts. Yet, consider, some of the most powerful tools of the analyst such as the impulse function (delta of Dirac), which has a value of infinity at zero yet zero elsewhere yet integrates over the domain evaluating to one, not a real function, and in the geometric context that the area under the point width at infinity equals one. That's much more directly reconcilable with the notion of the differential of the constant function, the sum of which over infinitely many points between zero and one equals the constant, than that instead those notions are not sound. Those tools are very regularly used.

Ross

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