Re: Does a potential infinity actually exist?

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On Aug 22, 5:08 pm, david petry <david_lawrence_pe...@xxxxxxxxx>
wrote:

> It seems that there is a lot of confusion about the notion of a "potential" infinity. Now I had always assumed that the people who were confused by this concept didn't know how to use Google, but then I tried myself to find a really clear explanation of the concept, and I was unable to. So I guess it's up to me to explain it.
>
> The idea of a potential infinity may be equated with the view of infinity held by Gauss, Poincare, and Weyl. The following quotes characterize this view:
>
> Gauss (paraphrased): "The notion of a completed infinity doesn't belong in mathematics; infinity is merely a figure of speech which helps us talk about limits"
>
> Poincare (quoted from Morris Klein): "Actual infinity does not exist. What we call infinite is only the endless possibility of creating new objects no matter how many exist already"
>
> Hermann Weyl: "... classical logic was abstracted from the mathematics of finite sets and their subsets .... Forgetful of this limited origin, one afterwards mistook that logic for something above and prior to all mathematics, and finally applied it, without justification, to the mathematics of infinite sets. This is the Fall and original sin of [Cantor's] set theory ...."

In post after post after post, you depend too much on quotes − pretty much argument by weight of authority.

> So the basic idea, as so eloquently stated by Weyl, is that infinite sets do not exist in exactly the same sense as the sense in which finite sets exist, and that mathematics should distinguish between the two senses. The language of classical mathematics (in any sense the reader may chose to interpret that phrase) does not make any distinction whatsoever between different senses of existence, and

> hence it impossible, or nearly so, to even talk about the potential existence of infinity within the language of classical mathematics. That seems to explain why there is so much confusion about the notion of potential infinity.

No, the problem is that so many people, such as you, intone over and over about a difference between potential and actual infinity, but you give only informal descriptions of certain intuitions and not formal, mathematical definitions or axioms regarding potential infinity.

If you would either give 'potential infinity' as a primitive and axioms with it or 'potential infinity' defined from primitives, then there might be something of specific mathematical interest there.

> The mathematical entities that have an "actual" existence are the things that we can actually compute. Thus, for example, we can actually compute an approximation of sqrt(2) accurate to seven digits, and hence such an approximation actually exists. Likewise, with a computer, we can compute a hundred digit approximation of sqrt(2), or even a million digit approximation, and so such approximations actually exist. In general, we can compute arbitrarily many digits of sqrt(2), but we cannot actually compute infinitely many such digits. That is, we cannot complete that task, and so the infinite string of digits representing sqrt(2) exactly does not actually exist. Instead, we say that it has only a potential existence, by which we mean that we can only actually compute arbitrarily accurate approximations to it.

Hilbert long ago distinquished between the contentual and the ideal, and since then mathematics has developed the notion of primitive recursion and classes of certain kinds of mathematical sentences such as to distinguish "degrees" of "distance" from primitive recursion. But you don't offer a mathematical theory to make rigorous your informal notion of potential infinity as approximation.

> What Gauss, Poincare and Weyl realized, but many mathematicians today fail to realize, is that we don't actually need infinity to do mathematics.

Depends on what "do mathematics" means. For certain people, mathematics is "done" axiomatically. And the axioms of set theory (which include the axiom of infinity) are efficient, intuitive, and easy to work with as an axiomatization of ordinary working mathematics. Perhaps there are finitistic axiomatizations of ordinary working mathematics, but we can't compare them with the axioms of set theory unless those finitistic axioms are presented to us. Of course, you don't present anything like that, as you are too busy repeating over and over your polemics and from your cache of quotations.

> That is certainly true about the mathematics of the real world (i.e. the mathematics used in physics and computer science, etc.) What we are actually interested in is the things that have an actual existence, and the things with a potential existence should be thought of as useful fictions or figures of speech which help us reason about the things that actually exist.

That is indeed the view of certain people who work in set theory and set theoretic axiomatized mathematics.

> In other words, the notion of infinity introduces no new theorems into mathematics; everything that can be said using the notion of infinity could also be said without it.

Could be SAID...perhaps. But can be PROVEN is another matter. When you provide some axioms and primitives that PROVE things about approximations and the like, THEN you'll have gotten past your currently over TWENTY YEAR rut of primarily intoning polemics over and over again.

> (Or equivalently, those assertions that absolutely require the axiom of infinity or the axiom of choice have nothing to tell us about the things that actually exist)

Even in the very sense of a "useful fiction" that you mentioned, the axiom of infinity DOES have a LOT to do with PROVING various theorems of ordinary working mathematics.

Granted, it would be welcome to have an axiomatization that proves ONLY what you take to be computational and scientific mathematical theorems and doesn't prove also all the stuff about infinities. I'd be very interested in studying such a thing (and I know there are proposed theories along such lines; so I'm not YET well informed about them only because my study time is limited and it has seemed to me that the finitistic proposals I've perused briefly are a lot more complicated − even in the syntax of the language − than set theory).

> To illustrate these points, consider the following two sentences:
>
> 1) sum (k=1..oo) 1/2^k = 1

2) Ap En Am (m>n) −> |sum (k=1..m) 1/2^k − 1| < 1/p

Here, 'A' and 'E' represent the universal and existential quantifiers, and the variables p,n,m are natural numbers.

The two sentences say exactly the same thing, but the first sentence invokes infinity, while the second doesn't. Most people would agree that the first sentence is far more readily comprehended, and so we see that we don't actually need infinity, but infinity is indeed a very useful figure of speech.

Again, you're going right past the difference between EXPRESSING in a language and PROVING in a theory. To PROVE things about limits of functions and such, we use an axiomatization that also happens to prove things about the fictional (or 'ideal' or whatever) infinities.
So, again, let us know when you have axioms and primitives to make the proofs but without infinities.

So how would we develop a set theory if we insist that infinity has only a potential existence? For starters, finite sets of integers are no problem. They have an exact representation as data structures in a computer, for example. The set of all integers is also not much of a problem. We can define some sense in which the set of integers {1..N} is an approximation to the set of all integers {1..oo}, and then we can say that the set of all integers has a potential existence (i.e. arbitrarily accurate approximations to it actually exist).

When you get to "the set of all integers" you resort to vagueness and posturing. I'd just like to know what is the syntax of your formal language, what are your primitives, and what are your axioms.

But what about the real numbers? The basic idea is that we have to find a way to approximate the set of real numbers by entities that actually exist. Let me outline one way in which the theory of real numbers can be developed within the constraints imposed by maintaining that infinity only has a potential existence.

Recall that with the usual topology, the set of real numbers has a countable neighborhood base. The elements in that neighborhood base could be taken to be the pairs of rational numbers, and we can certainly say that that neighborhood base has a potential existence in the same sense as the set of all integers does. Then, instead of classical real analysis, we would develop a theory of interval analysis (such a theory has indeed been developed). And it's a good bet that interval analysis is capable of providing us with all the tools we need for real world mathematics−−after all, in the real world, our measurements only yield real numbers within some interval;

> they never give us infinite precision.

As soon as you got the word "could", what followed was not a proposal but some ruminations about what might be a proposal.

I'm not against informal brainstorming. It's fine that one has informal notions and intutitions to propose, but it would help, at least me, to sustain any interest if I had some assurance that the ultimate objective is a rigorous, formal mathematical theory and not just layer after layer after layer of informal notions.

> So in conclusion, when we say that infinity has only a potential existence, we are making a distinction between different senses of the word "existence". Classical mathematics fails to make the distinction, and that failure pushes mathematics in the direction of make−believe, and that's a deficiency of classical mathematics.

First, even finitistic mathematical objects are abstractions. And aside from my point of view about that, Hilbert's notions about the contentual and the ideal are a vastly richer and vastly superior point of departure for explanation than your polemics. Second, when you come up with a theory (not just a bunch of polemical objections and a smattering of ruminations), then there might be something to talk about in regards to a challenge to set theory.

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