Re: how to list all of the real numbers

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* *From*: "Ross A. Finlayson" <raf@xxxxxxxxxxxxxxx>
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On Aug 23, 1:45 pm, Jonathan Hoyle <jonho...@xxxxxxx> wrote:

> On Aug 13, 3:32 pm, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxx> wrote:
>
> > Well−order the reals, i.e., biject the set of real numbers to an initial segment of an ordinal. Then, apply "Cantor's first" or nested intervals as it is sometimes called, that each sucessive element brackets the remaining interval. Then, besides whether c = Aleph_1, Aleph_2, ..., where it is said to be consistently equal to each of those with the undecideable continuum hypothesis, yet none of them, besides that, consider: does there not always exist each real number of a segment of the continuum in the remaining bracket/interval?
> >
> > That is where, if some initial segment of the ordinal O equivalent to c led to a degenerate interval, for example [0,0] containing only the point/scalar/number 0, then there is a question as to whether the continuum's cardinality is thus simply equivalent to a lesser cardinal than O's.
>
> It is not. You will always run into a degenerate interval at some limit ordinal prior to c, thus undoing your "proof".

Then, if it is a degenerate interval, that is, containing only a given point, with both "endpoints" of the "interval" being identical, there is no line to be drawn between them that is not coincident. So, then for some ordinal X less than O = c, any f(X+1) of the left or right endpoint sequence (that being a(X) or b(X)) of the nested intervals has exacly that value of that point, else there is no f(X+1).

That's kind of the point, in allusion to that trichotomy of cardinals does not hold where the reals are a set.

So, and here I think you refer to the paragraph and a half or so above illustrating that for any given infinite cardinal C > Aleph_1 that C > c and C < c, as opposed to illustrating that the reals between zero and one are listable in basically only one way, so that makes the point instead of breaking the point.

> > Similarly to how other considerations lead to that the real numbers have a cardinality greater than any given ordinal, it leads to an argument that they then as well have a cardinality less than any given ordinal to which they are equivalent. This was discussed further some year and so ago in "On well−ordering(s) of the reals, infinity."
>
> It was nearly two years ago, and yes it was in that discussion where your outline was proven to be faulty. I could go over your logic flaw again, but it was detailed pretty well at the time, and you might just simply re−read that posted topic to refresh your memory.

Yes I should re−read it.

> > Then, where a real may be a mark of a point, consider whether Hardy would accept that each mark of a point, or simply point, would represent a real number. Then, there is a question: how can there be marked each point on the unit interval? Cantor's nested intervals rreesult would have that any attempt to stipple or pock the line into existence would fail. Yet, drawing from mark to mark has that any point so marked is only as a result of a generative process that each new point in the course of the line is immediately adjacent to: the previous point.
>
> Incorrect. Even in Geometry, points on a line cannot be consecutive. Euclid's very first postulate prevents this: "Between any two distinct points a line can be drawn." This would not be possible if the points are adjacent; therefore, no two points can ever be adjacent in geometry.

Then, there would be some consideration of the definition of distinct points, which I would call definitely distinct, and indefinitely distinct points.

The continuum of real numbers, where any numbers between zero and one are real numbers else they wouldn't be between zero and one, including the nilpotent infinitesimal iota and iota−sums and iota−multiples, such that infinitely many iota's in multiple equal exactly one, leads to reformulation of the infinitesimal analysis as infinitesimal analysis, where that is nonstandard and not necessarily Non−Standard. Infinity in numbers does actually exist. Then, where natural laws are expected to correspond to the behavior of mathematically defined entities, and furthermore be defined as mathematical entities, as in for example the universe containin itself and is infinite and construing physical objects as mathematical objects and funcitons between them as physical objects, eg force fields, the universe itself is an example of infinite set and powerset identical, the physical universe. So, in continuum analysis of physical objects, with the known incongruencies of a sort in the large and small (paradoxes as it were, effects), there should be considered that there are appreciable analytic effects in the meso−scale, observable reality, in basically the notion of the polydimensional point, and real numbers as beads on a string, structured at once as the complete ordered field.

> > It was mentioned that the reason that rigor/soundness in analysis is a perceived requirement is for the differentialists, or analysts. Yet, consider, some of the most powerful tools of the analyst such as the impulse function (delta of Dirac), which has a value of infinity at zero yet zero elsewhere yet integrates over the domain evaluating to one, not a real function, and in the geometric context that the area under the point width at infinity equals one. That's much more directly reconcilable with the notion of the differential of the constant function, the sum of which over infinitely many points between zero and one equals the constant, than that instead those notions are not sound. Those tools are very regularly used.

> The Dirac delta function is not a function of the Reals, since its value at 0 is infinite. It is a function of the Extended Reals, that is the reals plus an ideal point called "infinity", using the traditional lemniscate as its symbol. The Extended Reals (denoted Roo) are topologically equivalent to the circle, and has many nice properties, but some statements true in R become false in Roo and vice−versa. (For example, in R the functions e^x and 1/x are continuous and discontinuous respectively, but the reverse is true in Roo.) Moreover, some basic arithmetic niceties, such as R being closed over subtraction, fails in Roo.

Browsing some history books and reading etcetera it seems that much work in the notion of the infinite series and so on, particularly that which is used by the analyst today, was done in a framework (or lack thereof) of the intuitive and not necessarily intuitionistic (having both constructionist/constructivist and intuitionistic philosophies) where the framework that yielded a or the large part of analytic results today was built around a more pragmatic than formalist approach. Here, there is the impulse function, the unit impulse function integrating to one. Where the domain has an infinite value, say, oo, infinity, then you would have that the codomain contains that value. Here, the key feature of that value is that d(0) = oo, yet, actually I say that d(0) = oo/2. The point is that there is a symmetry between the infinity of the domain and the infinity of the codomain. Infinity is basically considered a constant, when it can be cancelled simply. Then, where there is the consideration that 1/x for x = 0, 1/0 = oo, 1/oo = 0, yet that e^oo would be considered a discontinuity, in the asymptotic analysis reveals that 2^x < 3^x < ... and log x < x < e^x etcetera, preserving trichotomy.

> Even within Roo, the Dirac delta function must be formally axiomitized since it is not provable from the axioms of Roo alone.

It's a valuable and correct tool, beautiful in that way: formalize it.

> Regardless, I fail to see the point you are trying to make about this function. The existence (or non−existence) Dirac delta's axiomitization is completely irrelevant to the topology or cardinality of R.
>
> Jonathan Hoylehttp://www.jonhoyle.com

No, they're inextricably related.

That's kind of the point, the set of real numbers must basically fulfill all the properties of the continuum of real numbers. As has been known for thousands of years in for example the paradoxes of Zeno, where Zeno does arrive and the limit is and can only be the sum, and for no finite input is the inductive argument of differentiation complete, there are a wide variety of correct and truth−preserving identities about the infinite that generally appall today's formalist. Form follows function.

Where these tools work and how exactly they do, in the real numbers, as functions and operations and so on of real numbers, the real numbers are richer than their general axiomatization would imply. Thus, where the continuum of real numbers has more features than a given axiom set adequate for other work, and much of it, supports, one would be remiss to suggest otherwise.

Line the real numbers up, map them to the naturals via the equivalency
function as one−sided points, in base one, two, three, or infinity.

Ross

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