# Re: how to list all of the real numbers

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- *From*: "Ross A. Finlayson" <raf@xxxxxxxxxxxxxxx>
- *Date*: Fri, 24 Aug 2007 15:32:22 −0700

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On Aug 23, 9:15 pm, Jonathan Hoyle <jonho...@xxxxxxx> wrote:

> On Aug 23, 7:04 pm, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxx> wrote:
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>> On Aug 23, 1:45 pm, Jonathan Hoyle <jonho...@xxxxxxx> wrote:
>>
>>> It is not. You will always run into a degenerate interval at
>>> some
>>> limit ordinal prior to c, thus undoing your "proof".
>>
>> Then, if it is a degenerate interval, that is, containing only a given
>> point, with both "endpoints" of the "interval" being identical, there
>> is no line to be drawn between them that is not coincident.
>
> That is correct.
>
>> So, then for some ordinal X less than O = c, any f(X+1) of the left
>> or right endpoint sequence (that being a(X) or b(X)) of the nested
>> intervals has exacly that value of that point, else there is no f(X+1).
>
> No, not necessarily. You are assuming that the failure occurs at some
> successor ordinal "X+1". You are forgetting the possibility that it
> will fail at some limit ordinal. Your proof holds for the former case
> but not for the latter.
>
>> That's kind of the point, in allusion to that trichotomy of cardinals
>> does not hold where the reals are a set.
>
> I don't understand this sentence. The trichotomy of cardinals states

that given any two cardinals k1 and k2, exactly one of the following three cases holds: k1<k2, k1=k2, or k1>k2. Only the Axiom of Choice is required for this trichotomy to hold true. It has nothing to do with the reals.

If both of O < c and O > c hold, cardinals are non−trichotomous or the reals aren't a set.

So, and here I think you refer to the paragraph and a half or so above illustrating that for any given infinite cardinal C > Aleph_1 that C > c and C < c, as opposed to illustrating that the reals between zero and one are listable in basically only one way, so that makes the point instead of breaking the point.

I don't following this at all. I know at least your conclusion is false, as there are an uncountable number of ways that the reals can be ordered.

It was nearly two years ago, and yes it was in that discussion where
your outline was proven to be faulty. I could go over your logic flaw
again, but it was detailed pretty well at the time, and you might just
simply re−read that posted topic to refresh your memory.

Yes I should re−read it.

I think you would do well to. Despite the fact that it shows your proof to be in error, I think it was an example of your best work mathematically and logically. With a little effort, your construction could be used as a proof for the existence of limit ordinals.

Various topics are covered.

> > > Incorrect. Even in Geometry, points on a line cannot be
> > > consecutive.
> > > Euclid's very first postulate prevents this: "Between any two
> > > distinct
> > > points a line can be drawn." This would not be possible if the
> > > points
> > > are adjacent; therefore, no two points can ever be adjacent in
> > > geometry.

> > Then, there would be some consideration of the definition of distinct
> > points, which I would call definitely distinct, and indefinitely
> > distinct points.

> I don't know what you mean by "definitely distinct" and "indefinitely
> distinct". But according to Euclid's 1st postulate, if any two points
> *are not the same*, then a line can be drawn between them. Any
> concept of "adjacent points" is provably inconsistent in geometry.

Euclid's postulates have long been accepted to define geometry.

> > The continuum of real numbers, where any numbers between zero and one
> > are real numbers else they wouldn't be between zero and one, including
> > the nilpotent infinitesimal iota and iota−sums and iota−multiples,
> > such that infinitely many iota's in multiple equal exactly one, leads
> > to reformulation of the infinitesimal analysis as infinitesimal
> > analysis, where that is nonstandard and not necessarily Non−Standard.

> <snip>
>
> This run−on sentence is ill−defined and not even well thought out.
> You need to break this into steps. By doing so, the errors will
> become more apparent.

No, it's readable.

> > Browsing some history books and reading etcetera it seems that much
> > work in the notion of the infinite series and so on, particularly that
> > which is used by the analyst today, was done in a framework (or lack
> > thereof) of the intuitive and not necessarily intuitionistic (having
> > both constructionist/constructivist and intuitionistic philosophies)
> > where the framework that yielded a or the large part of analytic

> results today was built around a more pragmatic than formalist approach.

Yes, much of early analysis lacked the rigor it has today. Even many of Euler's proofs were playgrounds of inconsistencies and unwarranted assumptions. When Weierstrass and Bolzano came upon the scene, they added the rigorous approach to analysis so that it would be on as firm a foundation as Geometry had always been.

I think there is more than one approach.

> Here, there is the impulse function, the unit impulse function integrating to one. Where the domain has an infinite value, say, oo, infinity, then you would have that the codomain contains that value. Here, the key feature of that value is that d(0) = oo, yet, actually I say that d(0) = oo/2.

On the Extended Real line, the equation oo/2 yelds oo. oo/2 is not separate number. It is an algebraic formula taking oo and dividing it by 2, yielding back oo. You can see why this must be by considering that the inverse of oo is 0. If oo/2 is a different number than oo, then its inverse must be as well. However, the inverse of oo/2 is simply 2*0 = 0.

> The point is that there is a symmetry between the infinity of the domain and the infinity of the codomain.

No, on the extended real line, there is a symmetry between oo and the number 0. And since 0 cannot be changed by multiplying it or dividing it by another non−zero number, the same must be true for its inverse.

> Then, where there is the consideration that 1/x for x = 0, 1/0 = oo, 1/oo = 0, yet that e^oo would be considered a discontinuity, in the asymptotic analysis reveals that 2^x < 3^x < ... and log x < x < e^x etcetera, preserving trichotomy.

On the extended real line, the algebra of the real numbers remains the same, with the following additions (I think I have them right):

1. oo + x = oo, for all finite x
2. oo − x = oo, for all finite x
3. oo * x = oo, for all non−zero x
4. oo / x = oo, for all finite x
5. x / oo = 0, for all finite x
6. oo ^ x = oo, for positive, finite x
7. oo ^ x = 0, for negative finite x
8. 0 ^ x = oo, for negative finite x

oo − oo, oo / oo, 0/0, etc. all remained undefined as expected. Some others defined 0 * oo to be 0, whilst others left it undefined. (It was never defined to be 1, since that would be inconsistent.)

As an aside, you can extend the real line with two infinities, +oo and −oo. This has a very different algebraic set, and it makes the real line topologically equivalent to a line segment, not a circle. You lose the inverse of 0 and the two infinities, but it allows you to define x^+oo and x^−oo, which you cannot do in the above construction.

There are a wide variety of reasonable and useful considerations of infinity in the numbers.

Regardless, I fail to see the point you are trying to make about this
function. The existence (or non−existence) Dirac delta's axiomitization is completely irrelevant to the topology or cardinality
of R.

No, they're inextricably related.

That's kind of the point, the set of real numbers must basically fulfill all the properties of the continuum of real numbers. As has been known for thousands of years in for example the paradoxes of Zeno, where Zeno does arrive and the limit is and can only be the sum, and for no finite input is the inductive argument of differentiation complete, there are a wide variety of correct and truth−preserving identities about the infinite that generally appall today's formalist. Form follows function.

I do not follow. Zeno's "paradox" is nothing more than an invalid

> argument, making false assumptions about the nature of infinity. How does an unsound argument invalidate a mathematical proof?

So, the limit is the sum?

>> Where these tools work and how exactly they do, in the real numbers, as functions and operations and so on of real numbers, the real numbers are richer than their general axiomatization would imply.

> What do you mean by "richer"? Do you mean to include infinitesimals into R? Doing so will make R no longer complete, and a basic definition of R is one that makes it a complete ordered field.

>> Line the real numbers up, map them to the naturals via the equivalency function as one−sided points, in base one, two, three, or infinity.

> Can you please give me an example of this "equivalency function"? For instance, which finite natural number maps to pi?

....

You would have that there would always be statements about the real numbers that, although true, you could never prove. I, instead, would have that there is some true set of real numbers about which all facts are provable. (There is no consistent and complete theory of the real numbers without infinity acknowledged as not well−founded.)

Geometry definitely plays a key role in the perception of the real numbers. Where the primary objects of Euclid's geometry, where Euclid's "Elements" was the standard text for some 2000 years or more, are points and lines, I see the primary objects of geometry as being points and the ultimate space in which they all reside, with lines, surfaces, etcetera, made of points, lines being certain collections of points invariant under certain transformations of the space. That is where the ultimate space of sorts has the variously singly− or doubly− infinite orthogonal vector basis. (Infinity equals negative one.)
Where "Non−Euclidean" geometries, Riemannian, Lobachevskian, etcetera, are quite Euclidean, are there any known "non−Euclidean" geometries that presume to describe geo−metry, "world measure", using different primary objects than Euclid's? Then, there would certainly be the requirement of defining the line in terms of points in a Euclidean geometry which certainly exemplifies perceived perfection in form.
That is to say, in such a "non−Euclidean" geometry, lines defined in

terms of points in a space would have most of their normally accepted and understood properties, with some few varying on a reconstruction of the number system having a dual structure of the real numbers.

(Q: Where do space−filling curves and smoothly infinitesimally connected curves fit in Euclidean geometry? A: They don't.)

Consider this notion of a geometry where the primary objects are points and the ultimate space of points, vis−a−vis, a set theory with primary empty and universal sets. That is where, there is an analogy between the empty set and 0 = (0,0,0, ...), and via various generative successions (eg ordinally, permutationally, etcetera), the universe is the space. There, all surfaces are defined as point sets.

Consider the closing few sentences of the Wikipedia article on geometry,

"The history of 'lost' geometric methods, for example infinitely near points, which were dropped since they did not well fit into the pure mathematical world post−Principia Mathematica, is yet unwritten. The situation is analogous to the expulsion of infinitesimals from differential calculus. As in that case, the concepts may be recovered by fresh approaches and definitions. Those may not be unique: synthetic differential geometry is an approach to infinitesimals from the side of categorical logic, as non−standard analysis is by means of model theory." −− http://en.wikipedia.org/wiki/Geometry

The notion of mapping each point in the unit interval to a natural integer according to the real number's total linear ordering, their "natural" ordering, where for integers n, m: n < m => EF(n) < EF(m), and EF(0) = 0 and EF(oo) = 1, would have an infinity that has much of its properties being as a scalar constant. Then, for example, compared to the prototypical natural infinity, to coin a term as it were (unit scalar infinity), there is half an infinity, simply oo/2, and EF(oo/2) = 1/2, and, for real r, 0 <= r <= 1, EF( r oo) = r. It's quite simple, EF(n) = n/d, with integer n in the domain from 0 to d, d −> oo. EF(0) = 0,lim_n−>oo EF(n) = 1, the image is dense in the unit interval. N E N.

So, there is much to consider with regards to the nonstandard in analysis and non−Euclidean in geometry, and to justify those explorations vis−a−vis standard formalizations of logical structures (i.e. in set theory), having actual reasons to reject the perceived false axioms (ZF is inconsistent) leads to impetus by the mathematical conscience to reanalyze foundations of theories of sets, numbers, and geometry, towards what has long been a goal: reformulation and reformalization of the underpinnings, the foundations, to some extent the dogma, of the science of mathematics.

Ross

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