Re: Infinite series question

Source: http://sci.tech−archive.net/Archive/sci.math/2007−08/msg04207.html

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- *From*: Robert Israel <israel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>
- *Date*: Fri, 24 Aug 2007 19:21:02 −0500

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jane <jane1806@xxxxxxxxxx> writes:

> jane <jane1806@xxxxxxxxxx> writes:
>
> > Suppose, x_i in R, 0 < x_i < 1, sum_i x_i = oo,
>
> and
>
> > A_n = sum_{i <= i <= n} x_i, so that A_n −> oo.
> >
> > B_n = sum_{1 <= i <= n} x_i exp[−A_n * xi].
> >
> > Are there any known results under which
>
> conditions on
>
> > (x_i), we have that B_n −> 0 as n −> oo ?
>
> I forgot to mention in the previous post, that it
>
> is also known that lim
>
> x_i = 0 and i −> oo.

It's true if x_i > c i^(−p) for constants c > 0 and 0 < p < 1/2.
A_n > c int_1^(n+1) t^(−p) dt = c/(1−p) ((n+1)^(1−p) − 1) > b n^(1−p)
for n sufficiently large, if 0 < b < c/(1−p).

Since ln(t)/t is decreasing for large t, for n
sufficiently large we have
2 ln(A_n)/A_n < 2 (ln(b) + (1−p) ln(n))/(b n^(1−p)) <
c n^(−p) < x_i for
1 <= i <= n.

Then exp(−A_n x_i) <= exp(−2 ln(A_n)) = A_n^(−2) so
B_n <= sum_{i=1}^n x_i A_n^(−2) = A_n^(−1) −> 0 as n
−> infinity.

On the other hand, for x_i = i^(−1/2) we have
0 < A_n x_i <= 2 sqrt(2) for i >= n/2, and then
B_n >= A_n exp(−2 sqrt(2)) −> infty.
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This is helpful anyway, but quite not but i was initially looking for.
I was looking for some good convergence of the sum

C_n = sum_{1<=i<=n} x_i * prod_{1<=j<=n}(1−x_i * x_j).

I just used the inequality 1−t <= exp(−t) and then hoping for already good
convergence of the sum

B_n = sum_{1 <= i <= n} x_i exp[−A_n * xi] posted the question above, but
that didn't help according to you answer.

Do you think, is it possible to give some weaker conditions on x_i so that
for new C_n defined above we can get that lim_n C_n under the same
conditions
0 < x_i < 1, x_i −> 0, sum x_i = oo ?

Not much weaker.

If 0 < t < a < 1, exp(−ct) <= 1 − t where exp(−ca) = 1 − a
(i.e. c = −ln(1−a)/a), so if all x_i <= sqrt(a),
C_n >= sum_{i=1}^n x_i exp(−c A_n x_i)
Then again x_i = b i^(−1/2) (where 0 < b < 1) would have
A_n x_i <= some positive constant for i >= n/2, and
C_n >= k A_n (for some positive constant k)
−> infinity as n −> infinity.
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