chromatic number of a Hilbert space

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The problem of the chromatic number of a euclidean plane asks for the minimum number of colours needed to colour the plane so that any two points one unit distance apart are coloured differently.

In other words, an admissible colouring of (the normed space) R^2 is a mapping
f: R^2 −> N
such that if ||u−v|| =1, then f(u) != f(v) for all u, v in R^2.

#f(R^2) is then the number of colours used, and one seeks to minimize this for admissible colourings. As a recall, at least four colours are needed, but seven colours are enough.

Above, one could replace 'R' by L^2([0,1]), the separable infinite−dimensional complex Hilbert space of square−integrable functions on [0, 1]; also, N would be replaced by a colour−space such as R, the set of reals.

If {e_i}_{i in N} is a Hilbert base of L^2([0,1]), then for i !=j,
< (e_i − e_j)/sqrt(2) , (e_i − e_j)/sqrt(2) > = 2/(sqrt(2) * sqrt(2)) = 1.
<., .> is the inner product on the Hilbert space.

So we have aleph_0 points (the e_i/sqrt(2)) all at unit distance from each other.
So for this Hilbert space, at least aleph_0 colours are needed.

At the moment, I don't know whether an admissible colouring using aleph_0 colours exists.

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