

# Re: Coding of ordered pairs

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lwalke3@xxxxxxxxxx wrote:

It appears what HdB wants to define the ordered pair of two natural numbers without relying on ZFC or any other set theory at all. In other words, he wants to avoid sets and set notation and instead define the ordered pair only in the language of PA (with its primitives zero, successor, and "is a natural number") rather than the language of ZFC (with its primitive "is an element of"), in order to show how many results of mathematics] can be developed without set theory.

Quite close. My toy (bit mapped) set theory gives the following result for the Kuratowski ordered pair:

$$\{\{a\},\{a,b\}\} = \{2^a, 2^a + 2^b\} = 2^{(2^a)} + 2^{(2^a + 2^b)} \\ = 2^{(2^a)} \cdot (1 + 2^{(2^b)})$$

A very "sparse" result (as with the von Neumann ordinals in bit mapped set theory). Clearly, much more "dense" representations are possible.

Han de Bruijn

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