Re: Convex Functions

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- *From*: Maury Barbato <mauriziobarbato@xxxxxxxx>
- *Date*: Tue, 11 Sep 2007 11:12:08 EDT

Robert Israel wrote:

> quasi <quasi@xxxxxxxx> writes:
>
>> On Sat, 08 Sep 2007 18:23:00 EDT, Maury Barbato
>> <mauriziobarbato@xxxxxxxx> wrote:
>>
>>> Remembering that a function f:R^n−>R is said
>>> to be increasing if for every x, y in R^n, with
>>> x_i >= y_i for every i in {1,...,n}, we have
>>> f(x)>=f(y), we could have the following
>>>
>>> Let f:R^n−>R be an increasing function such that
>>> for every x,y in R^n we have
>>>
>>> f((x+y)/2)<=[f(x)+f(y)]/2.
>>>
>>> then f is convex.
>
> In fact, any function that is locally bounded above
> and midpoint−convex
> is convex.
>
> Proof: suppose f is not convex. There exist x, y,
> and t with
> 0 < t < 1 and f(t x + (1−t) y) > t f(x) + (1−t) f(y).
> For convenience
> I'll take t x + (1−t) y = 0. Now f(s x + (1−s) y) <=
> s f(x) + (1−s) f(y)
> for all dyadic rationals s = j/2^n, j = 0 .. 2^n. So
> there is
> a sequence u_n −> 0 with u_n <= b < f(0). Then for
> any positive integer m,
> f(0) <= (1 − 2^(−m)) f(u_n) + 2^(−m) f((1−2^m) u_n)

so f((1−2^m) u_n) >= (2^m − 1) (f(0) − f(u_n)) + f(0)

>= (2^m − 1) (f(0) − b) + f(0)
For any N and epsilon > 0, we can take m large enough
so
(2^m − 1) (f(0) − b) + f(0) > N,
and then n large enough so || (1 − 2^m) u_n || <
epsilon. This shows
that f is unbounded above in every neighbourhood of
0.
−−
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Ohh, what a wonderful proof!!! I had a job trying to
understand it, but is is worth the trouble! Superb!
My Best Regards,
Maury
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