

## Re: Probability of exceeding a specific value

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- *From:* Robert Israel <[israel@xx](mailto:israel@xx)>
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Randy Poe <[poespanm-trap@xxxxxxxxxx](mailto:poespanm-trap@xxxxxxxxxx)> writes:

On Sep 18, 11:31 am, grosu <[gr...@xxxxxxxxxxxxxxxx](mailto:gr...@xxxxxxxxxxxxxxxx)> wrote:

The price is not normally distributed.

Not by assumption. But it turns out that it is.

The initial price is known to be 10\$.  
Then, the price changes every time step, with the change in the price being normally distributed, that is the Delta of the price. But, the price can go down as well as up.

And the price at step  $k$  is  $10 +$  a sum of  $k$  normals with zero mean and variance 1. Thus, the price at step  $k$  is normally distributed with mean 10 and variance  $k$ .

I am not interested in the price after  $k$  time steps. I am interested in the probability of the price exceeding 13\$.

There's no single answer to that. That probability increases in time. It is  $P(Z > 3/\sqrt{k})$ . The probability that \$13 will be exceeded eventually is 100%.

and the time expected for this to happen.

I can't see an easy way to approach this, but it could be approximated numerically by evaluating  $P(Z > 3/\sqrt{k})$  for a large finite number of values of  $k$ .

Re: Probability of exceeding a specific value

If  $u(p)$  is the expected number of steps for the price to exceed  $10 + p$  (starting with price 10), we have

$$u(p) = \begin{cases} 0 & \text{for } p \leq 0 \\ 1 + \int_0^{\infty} f(p-x) u(x) dx & \text{for } p > 0 \end{cases}$$

where  $f(x)$  is the probability density function for the normal distribution with mean 0 and variance 1. I don't know if this integral equation has a closed-form solution, but you could probably do fairly well using finite sums to approximate the integral.

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