Re: Probability of exceeding a specific value

Source: http://sci.tech−archive.net/Archive/sci.math/2007−09/msg04225.html

* *From*: David Bernier <david250@xxxxxxxxxxxx>
* *Date*: Wed, 19 Sep 2007 09:27:33 −0400

matt271829−news@xxxxxxxxxxx wrote:

> On Sep 19, 11:35 am, grosu <gr...@xxxxxxxxxxxxx> wrote:
>
>> Let's try out something:
>>
>> The probability of price equal 13$ is increasing with time.
>
> The probability that the price equals $13 is zero at all times.
> Perhaps you mean the probability of the price being greater than or
> equal to $13?
>
>> However, the mean of this probability is still zero.
>> Assuming the price is changing in constant time intervals of dt, the variance
>> of the probability of the price after n time intervals is n − p(x,t)=
>> p(x,ndt)=N(10,n), where x is the price.
>
> The variance of the price distribution after n time intervals is just
> n. Variance is additive.
>
>> If this assumption is correct then, the probability of the price to be 13$, in
>> any time, is the sum of the probabilities over time. Thus, P(x)= N(10,n),
>> where N is the normal (exponential) distribution funcion.
>
> This doesn't make any sense: you're adding things that can't be
> meaningfully added. The sort of thing that I think you're trying to do
> won't work anyway because the price distributions at different times
> aren't independent.
>
>> I'm not sure about this, since after substituting x=3, I got probability higher
>> than 1 after 35 time intervals. Any comments/thoughts?

> By the way, you realise that model you're using gives a non−zero probability for the price being negative? It does not take too many steps for this probability to become significant.
>
> Since neither of my earlier posts with the exact formulas for the continuous (i.e. Brownian motion) case have shown up in Google Groups, here it is YET again, in case it is of any use. Sorry to all about the repetition.
>
> "Given that the expectation is infinite, I imagine that the OP might find the distribution of the first time to hit $13 more useful. Then I think the discrete problem (discrete jumps of N(0,1)) is different from the continuous case, even though the price distributions at some given time are the same. Basically because with the continuous case you've got the chance of hitting $13 and then falling back between timesteps.
>
> Failing to see how to find a closed form for the discrete case, I looked at the continuous case, translated to start at zero and looking to hit k (so in the original problem k = 3). I got the probability density function of the first time to hit k to be
>
> 1/Sqrt(2*pi) * k * t^(−3/2) * Exp(−k^2/(2*t))
>
> with the cumulative distribution being
>
> 1 − Erf(k/Sqr(2*t))
>
> Ans, as, erm, expected the expected value of t looks to be infinite.
>
> Comparing this with simulations of the discrete case there is a fair discrepancy."

I looked for the pdf for the first time a random walker on Z would reach 3 starting at 0. I didn't find any exact formula. What I found is that "stopping time" is often used for the first time a certain state is reached. Also, I saw a discussion of the "gambler's ruin" problem. If the game is fair, and players A and B start with $m and $n, m,n>0, betting $1 each time, then if I remember correctly the probability that the one with $n is eventually ruined is m/(m+n), and symmetrically n/(m+n) for the one with $m. If m and n are finite, the stopping time (when ruin happens to either A or B) has a finite expected value. If A has infinite money, then B is ruined with probability 1; the expected time is infinite.

An interesting variation is the distribution (pdf) for the first time a random walker returns to his starting place, on Z. The expectation was said to be infinite. The formulation was equivalent to a random walk on Z, but formulated as follows: "Suppose we toss a fair coin. Let T (positive) be first toss number when we have equal numbers of heads and

tails." Clearly, T is even. It was mentioned that E(T) = oo.
I don't know the distribution of T, but the probability
of n heads and n tails in 2n tosses is C(2n, n)* 2^(−2n).
The pdf of T=2n is just A(2n)/(2^(2n)) , where
A(2n) is the number of random walk paths of length
2n steps where the starting point was visited only
once.

David Bernier

.