

Re: Rational numbers, irrational numbers: each dense in real numbers

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On Sep 20, 10:17 pm, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxxx> wrote:

A well ordering is an ordering relation on elements of a set such that each subset of the set has a least element by the ordering.

Close.

R is a well ordering on S \leftrightarrow (R is a strict total ordering on S & every nonempty subset S has an R-least member).

(At least I consider each element to be in the universe,

Obviously any object mentioned in a theory is a member of any universe of a model of the theory.

and when
collections are defined by their elements

The axiom of extensionality stipulates that a set is determined only by its elements, if that's what you mean.

and have the element-of and
subset defined that they're sets.)

I don't know what that is supposed to mean.

What's your point?

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Your wrote:

"In ZFC, with standard definitions of the real, rational, and irrational numbers, let p_i be an irrational number between zero and one for i from a suitably large well-ordered index set X . With the well-ordering of the index set, let the i 'th element p_{i+1} be an irrational number between zero and p_i , where $i+1$ is the least element of the well-ordering $X \setminus i$, that is defined to equal X_{i+1} ."

Take it step by step:

Let R be a well ordering of X non-empty.

Let p be a function from X into $\{r \mid r \text{ is irrational} \ \& \ r \in (0, 1)\}$.

After that, your formulation is incoherent (but it's still not the fatal flaw, since we could still fix your incoherent formulation and find a deeper mistake): You say " $i+1$ is the least element of the well-ordering $X \setminus i$ ". In other words, you're defining a 1-place operation on X . But the notation ' X_i ' indicates that X itself is also a function. But you've not said what function X is. Moreover, you say "setminus i " when you must mean "setminus (i.e. complement) $\{y \mid y \text{ R-less than } i\}$ ".

So we might address that mess this way (and from now on instead, of 'R-less' and things like that, I'll just say '<' etc. instead of 'R-less' as R is understood by context:

What you want is to say what " $i+$ " is for any element i of X .

So, since R is a well ordering of X , there is a successor relation in X (whether or not it's the ordinary successor relation of the ordinals $-i \cup \{i\}$ - is not crucial; what's important is that we can define 'R-successor' and thus have our successor relation on X on that basis. (By context, we can just say 'successor' instead 'R-successor')

So $i+ =$ the successor of i .

Then, let c be the least member of X . So $p(c)$ is some irrational in $(0, 1)$.

Then, for any i in X , $p(i+)$ is some irrational in $(0, 1) \setminus \{p(y) \mid y < i\}$.

And we'll add a stipulation about p that you mentioned previously: For every $i+$, we have $p(i+) < p(i)$.

Hint here: What is missing at this point?

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Then you say, "There are uncountably many irrational numbers less than each p_i and greater than zero".

Yes, for any $r > 0$, there are uncountably many irrational numbers in any interval $(0, r)$. But, back to the hint, something is not accounted for in your construction so far.

Then you define P as the range of p .

Okay, but think about the hint again now.

Then you say, "There exists a rational number q_i between p_i and p_{i+1} "

Yes, for any i in X , there is a rational number between $p(i)$ and $p(i+1)$.

You continue, "For each of the irrational p_i 's, there thus exists at least one unique rational q_i between p_i and p_{i+1} , and infinitely many."

Uncountably many indeed.

Then you say, "Let the ordered pair (p_i, q_i) be an element of a function, as a set, from P to Q ."

I'd make that:

Let q be a choice function on P such that $q(p(i)) =$ an irrational between $p(i)$ and $p(i+)$.

So $p(i+) < q < p(i+)$.

If you want this to be on X , then:

Let q' be a choice function on X such that $q'(p(i)) =$ an irrational between $p(i)$ and $p(i+)$.

Then you say, "If there is an uncountable set P of irrational numbers in $(0,1)$ "

IF. Is P uncountable? P is the range of p . Go back to the hint now!

You finish, "then there is a 1-to-1 function defined by the set $\{(p_i, q_i), i \in X\}$ from uncountable P to a subset of Q the rational numbers"

I don't even have to check whether the function is 1-1. You just need to go back to the hint, which will lead you to reexamine your claim that P is uncountable.

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