Re: Rational numbers, irrational numbers: each dense in real numbers

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- *From*: MoeBlee <jazzmobe@xxxxxxxxxxx>
- *Date*: Mon, 24 Sep 2007 11:48:05 −0700

On Sep 22, 5:53 am, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxx> wrote:

> On Sep 21, 1:12 pm, MoeBlee <jazzm...@xxxxxxxxxxx> wrote:
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>> On Sep 20, 10:17 pm, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxx> wrote:
>>
>>> A well ordering is an ordering relation on elements of a set
>>> such that
>>> each subset of the set has a least element by the ordering.
>>
>> Close.
>>
>> R is a well ordering on S <−> (R is a strict total ordering on S &
>> every nonempty subset S has an R−least member).
>>
>>> (At least I consider each element to be in the universe,
>>
>> Obviously any object mentioned in a theory is a member of any universe
>> of a model of the theory.
>>
>>> and when
>>> collections are defined by their elements

The axiom of extensionality stipulates that a set is determined only by its elements, if that's what you mean.

> and have the element−of and subset defined that they're sets.)

I don't know what that is supposed to mean.

> What's your point?

Your wrote:

"In ZFC, with standard definitions of the real, rational, and irrational numbers, let p_i be an irrational number between zero and one for i from a suitably large well−ordered index set X. With the well−ordering of the index set, let the i'th element p_{i+1} be an irrational number between zero and p_i, where i+1 is the least element
of the well−ordering X_i setminus i, that is defined to equal X_{i +1}."

Take it step by step:

Let R be a well ordering of X non−empty.

Let p be a function from X into {r | r is irrational & r e (0 1)}.

After that, your formulation is incoherent (but it's still not the fatal flaw, since we could still fix your incorehent formulation and find a deeper mistake): You say "i+1 is the least element of the well−ordering X_i setminus i". In other words, you're defining a 1−place operation on X. But the notation 'X_i' indicates that X itself is also a function. But you've not said what function X is. Moreover, you say "setminus i" when you must mean "setminus (i.e. complement) {y+ | y R−less than i}.

So we might address that mess this way (and from now on instead, of 'R−less' and things like that, I'll just say '<' etc. instead of 'R−less' as R is understood by context:

What you want is to say what "i+" is for any element i of X.

So, since R is a well ordering of X, there is a successor relation in X (whether or not it's the ordinary successor relation of the ordinals − i u {i} − is not crucial; what's important is that we can define 'R−successor' and thus have our successor relation on X on that basis. (By context, we can just say 'successor' instead 'R−successor)

So i+ = the successor of i.

Then, let c be the least member of X. So p(c) is some irrational in (0 1).

Then, for any i in X, p(i+) is some irrational in (0 1)\{p(y) | y < i}.

And we'll add a stipulation about p that you mentioned previously: For every i+, we have p(i+) < p(i).

Hint here: What is missing at this point?

Then you say, "There are uncountably many irrational numbers less than each p_i and greater than zero".

Yes, for any r>0, there are uncountably many irrational numbers in any interval (0 r). But, back to the hint, something is not accounted for in your construction so far.

Then you define P as the range of p.

Okay, but think about the hint again now.

Then you say, "There exists a rational number q_i between p_i and p_{i +1}"

Yes, for any i in X, there is a rational number between p(i) and p(i +1).

You continue, "For each of the irrational p_i's, there thus exists at least one unique rational q_i between p_i and p_{i+1}, and infinitely many."

Uncountably many indeed.

Then you say, "Let the ordered pair (p_i, q_i) be an element of a function, as a set, from P to Q."

I'd make that:

Let q be a choice function on P such that q(p(i)) = an irrational between p(i) and p(i+).

So p(i+) < q < p(i+).

If you want this to be on X, then:

Let q' be a choice function on X such that q(p(i)) = an irrational between p(i) and p(i+).

Then you say, "If there is an uncountable set P of irrational numbers in (0,1)"

IF. Is P uncountable? P is the range of p. Go back to the hint now!

You finish, "then there is a 1−to−1 function defined by the set {(p_i, q_i), i E X} from uncountable P to a subset of Q the rational numbers"

I don't even have to check whether the function is 1−1. You just need to go back to the hint, which will lead you to reexamine your claim that P is uncountable.

MoeBlee

No. I don't mean "setminus (i.e. complement) {y+ | y R− less than i}", which is obscure and immediately contextual,

You may not mean it, but it's not obscure; it's perfectly well defined. And I don't know what you mean by "immediately contextual" other than that, of course, it depends on R.

Anyway you said, ""i+1 is the least element of the well−ordering X_i setminus i"

I explained why that is mixed up. First of all, you haven't said what way X is a function that takes arguments i.

I mean p_i.

So it was i now it's p_i. Okay. But it's still incoherent as you haven't said in what way X is a function that takes arguments i.

> It is not "Then, for any i in X, p(i+) is some irrational in (0 1)\ {p(y) | y < i}", instead "Then, for any i in X, p(i+) is some irrational in (0 1)\ (p_i, 1)".

Later I made another stipulation that, on my merely cursory glance, reduces to just what you said (actually going in downwards instead of upwards, but that's not of any consequence). This point does not matter much, since the error in your argument is even more basic.

> What's your point?

I gave you the hint at the EXACT point where your fundamental error (not just some problem in notation or formulation, but rather a fundamental misconception) begins.

I'll state it even more specifically this time (but I'll still leave it for you to reason out the rest of it):

When you define p(i+) in terms of p(i), you're assuming that p(i) is defined. But for p(i) to be defined, what must be established about i?

Let's have you concentrate just on that one question right now. Tell me what you come up with.

MoeBlee

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