Re: ways to define harmonic functions

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* *Date*: Sat, 29 Sep 2007 11:13:21 −0000

On Sep 29, 6:21 pm, "G. A. Edgar" <ed...@xxxxxxxxxxxxxxxxxxxxxxxxxxx>
wrote:

> The term "harmonic function" means something else in mathematics.

Thank you for this correction.

> * define log z = integral(1 to z) dx/x, then exp is the inverse of it.
> [also works, with care, for complex z]
> * define arctan z = integral(0 to z) dx/(1+x^2), then tan is the
> inverse of it. sin and cos then defined by identities from tan.

Thank you for this explanation and help. It is very clear. I
appreciate it.

Dan

On Sep 29, 6:21 pm, "G. A. Edgar" <ed...@xxxxxxxxxxxxxxxxxxxxxxxxxxx>
wrote:

> In article <1191031460.581720.164...@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
> Daniel J. Greenhoe <dgreen...@xxxxxxxxx> wrote:
>
>> There are at least three ways to define harmonic functions such as
>> sine, cosine, and the complex exponential:
>> 1. as relations between sides and angles of triangles in plane
>> geometry
>>
>> 2. as solutions of second order homogeneous differential equations
>> with certain initial conditions as in

D^2 cos + cos = 0 with init.cond. cos(0)=1 [Dcos](0)=0
D^2 sin + sin = 0 with init.cond. sin(0)=0 [Dsin](0)=1
D^2 expi + expi = 0 with init.cond. expi(0)=1 [Dexpi](0)=i

3. as polynomials in a Taylor expansion

Does anyone know of any other definitions of harmonic functions on any other mathematical structures?

Many thanks in advance,
Dan Greenhoe

The term "harmonic function" means something else in mathematics.
But as I understand you want to know various ways to define
sine, cosine, exp.

You have given three of them.

A fourth is in terms of integrals...this can be found in some calculus
textbooks...It works like this:

* define log z = integral(1 to z) dx/x, then exp is the inverse of it.
[also works, with care, for complex z]

* define arctan z = integral(0 to z) dx/(1+x^2), then tan is the
inverse of it. sin and cos then defined by identities from tan.
[Or I suppose you could directly use integral for arcsin.]

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