Re: Another algebra question

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- *From*: magidin@xxxxxxxxxxxxxxxxx (Arturo Magidin)
- *Date*: Sun, 30 Sep 2007 20:44:30 +0000 (UTC)

In article <1191170849.443132.285730@xxxxxxxxxxxxxxxxxxxxxxxxxxx>,
Snis Pilbor <snispilbor@xxxxxxxxx> wrote:

> On Sep 30, 9:10 am, quasi <qu...@xxxxxxxx> wrote:
>
>> On Sun, 30 Sep 2007 01:55:37 −0700, Snis Pilbor <snispil...@xxxxxxxxx>
>> wrote:
>>
>>> another thing I am stumped on. Suppose R is a ring in a field
>>> F, and
>>> R is integrally closed in its field of fractions. I want to show
>>> that
>>> an element x in F is integral over R (that is, it's annihilated
>>> by
>>> some monic polynomial with coefficients in R), iff its
>>> minimal
>>> polynomial has coefficients in R.
>>
>> Hints:
>>
>> Suppose x is integral over R. Let K be the quotient field of R.
>>
>> (1) Let p be the minimal polynomial of x over K. By definition, p is
>> monic and the coefficients of p are in K.
>>
>> (2) Argue that the coefficients of p are integral over R [think about
>> symmetric polynomials].
>>
>> (3) But R is integrally closed, so ...
>>
>> quasi
>
> Interesting, but this only shows that the minimal polynomial of x over
> K are elements of R. Couldn't x have a different minimal polynomial
> over F?

Since x is assumed to be an element of F, its minimal polynomial over F is X − x. Probably not the same one as over K (unless x happens to lie in K as well)

But... So what? "Minimal polynomial" above refers to the minimal over R (i.e., over K), not over F.

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"It's not denial. I'm just very selective about
what I accept as reality."
−−− Calvin ("Calvin and Hobbes" by Bill Watterson)
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