Re: density of the range of an integer polynomial

Source: http://sci.tech−archive.net/Archive/sci.math/2007−10/msg00899.html

- *From*: quasi <quasi@xxxxxxxx>
- *Date*: Thu, 04 Oct 2007 23:56:49 −0400

On Fri, 05 Oct 2007 03:40:39 GMT, Gerry Myerson
<gerry@xxxxxxxxxxxxxxxxxxxxxxxxx> wrote:

> In article <r18bg3hem590879ct2jidonu3n4p41h2hp@xxxxxxx>,
> quasi <quasi@xxxxxxxx> wrote:
>
>> On Tue, 02 Oct 2007 23:13:13 −0500, Robert Israel
>> <israel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:
>>
>>>> (2) For an integer polynomial, what densities
>>>> are possible? Is every
>>>> rational number in the interval [0,1]
>>>> achievable as a density? Can a
>>>> density be irrational?
>>>
>>> For density m/n, try f(x,y) = x + n y + (x^9 + y^9)
>>> prod_{j=1}^m (x − j).
>>
>> I can verify the correctness of the above experimentally, but I don't
>> see why it works.
>
> It's pretty clear you get all the numbers congruent 1, 2, ..., m mod n
> by taking x = 1, 2, ..., m and y running through Z. If x is anything
> else then the y^9 term kicks in and gives terms with zero density.
>
>>> For the irrational density 1 − 6/pi^2, try
>>> f(v,w,x,y,z) = v (w^4 + x^4 + y^4 + z^4 + 2)^2
>>
>> For this one, I'll guess that it's related to Buffon's Needle −−
>> either that or there's an algebraic tie to an infinite series, but I
>> don't have a clear strategy. Moreover, it's not even that easy to

> > verify experimentally.
>
> The polynomial generates the integers that are not squarefree,
> that is, it generates those positive integers that have a square
> factor exceeding 1. It known (see any good intro number theory
> text) that the squarefree numbers have density pi^2 / 6.

Thanks −− mystery resolved.

quasi
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