

set notation

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Students commonly think that the notation " $\{\emptyset\}$ " denotes the empty set. Many secondary school teachers think this, too.

Mistakes in reading math notation occur because the reader's understanding of the notation system is different from the author's. The most common bits of the symbolic language of math have fairly standard interpretations that most mathematicians agree on most of the time. Students develop their own non-standard interpretation for many reasons, including especially cognitive dissonance from ordinary usage and ambiguous statements by teachers.

I believe (from teaching experience) that when a student sees " $\{1, 2, 3, 5\}$ " they think, "That is the set 1, 2, 3 and 5". The (incorrect) rule they follow is that the curly braces mean that what is inside them is a set. So clearly " $\{\emptyset\}$ " is the empty set because the symbol for the empty set is inside the braces.

However, "1, 2, 3 and 5" is not a set, it is the names of four integers. A set is not its elements. It is a single mathematical object that is different from its elements but determined exactly by what its elements are. The correct understanding of set notation is that what is inside the braces is an expression that tells you what the elements of the set are. This expression may be a list, as in " $\{1, 2, 3, 5\}$ ", or it may be a statement in setbuilder format, as in " $\{x \mid$

$x \in \{1, 2, 3, 5\}\}$ ". According to this rule, " $\{\emptyset\}$ " denotes the singleton set whose only element is the empty set.

This posting is based on the belief that that mathematical notation has a standard, (mostly) agreed-on interpretation. I made this attitude explicit in the second paragraph. Teachers rarely make it explicit; they merely assume it if they think about it at all.

The student's interpretation is a natural one. (Proof: So many of them make that interpretation!) Did the teacher tell the student that math notation has a standard interpretation and that this is not always what an otherwise literate person would expect? Did the teacher explain the specific and rather subtle rule about set notation that I

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described two paragraphs above? If not, the student does not deserve to be ridiculed for making this mistake.

Many people who get advanced degrees in math understood the correct rule for set notation when they first learned it, without having to be told. Being good at abstract math requires that kind of talent, which is linguistic as well as mathematical. Most students in abstract math classes are not going to get an advanced degree in math and don't have that talent. They need to be taught things explicitly that the hotshots knew without being told. If all math teachers had this attitude there would be fewer people who hate math.

PS: My claim about how students think that leads them to believe that " $\{\emptyset\}$ " denotes the empty set is a testable claim. There are many reports in the math ed literature from investigators who have been able to get students to talk about what they understand, for example, while working a word problem, but I don't know of any reports about my assertion about " $\{\emptyset\}$ ".