

Re: ranges of integer polynomials

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Just to give an update ...

Of the 12 problems I posed in this thread, 3 of them — problems 1, 6, 8, have now been resolved.

There are 9 problems still left from the original group, and I believe that they are all within reach.

For reference, here are the problems not yet resolved:

Let f be an integer polynomial, possibly multivariate, and let $\text{range}(f)$ denote the range of f for all integer inputs.

problem (2):

Can $\text{range}(f) = \{x^2 \mid x \in \mathbb{Z}\} \cup \{-x^2 \mid x \in \mathbb{Z}\}$?

problem (3):

(a) Can $\text{range}(f) = \{x^2 \mid x \in \mathbb{Z}\} \cup \{2x \mid x \in \mathbb{Z}\}$?

(b) Can $\text{range}(f) = \{x^2 \mid x \in \mathbb{Z}\} \cup \{2x \mid x \in \mathbb{N}\}$?

problem (4):

(a) Can $\text{range}(f) = \{x^2 \mid x \in \mathbb{Z}\} \cup \{x^3 \mid x \in \mathbb{Z}\}$?

(b) Can $\text{range}(f) = \{x^2 \mid x \in \mathbb{Z}\} \cup \{x^3 \mid x \in \mathbb{N}\}$?

problem (5):

Must at least one of the sets

$(\text{range}(f) \cap \mathbb{N})$

$(\text{range}(-f) \cap \mathbb{N})$

have a density, as a subset of \mathbb{N} ?

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problem (7):

If $\text{range}(f)$ is a subset of \mathbb{N} , must $\text{range}(f)$ have a density, as a subset of \mathbb{N} ?

problem (9):

(a) Can $\text{range}(f)$ be the set of all integer non-cubes?

(b) Can $\text{range}(f)$ be the set of all positive integer non-cubes?

problem (10):

(a) For which positive integers $k > 2$, if any, can the set of all integer non- k 'th powers be realized as $\text{range}(f)$?

(b) For which positive integers $k > 2$, if any, can the set of all positive integer non- k 'th powers be realized as $\text{range}(f)$?

problem (11):

Can $\text{range}(f) =$ the set of squarefree positive integers?

problem (12):

(a) If f is not constant and $\text{range}(f)$ contains zero as a least element, must there exist an integer polynomial g such that $\text{range}(g) = \text{range}(f) \setminus \{0\}$?

(b) If f is not constant and $\text{range}(f)$ is a subset of \mathbb{N} , must there exist an integer polynomial g such that $\text{range}(g) = \text{range}(f) \cup \{0\}$?

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