# Re: Two results of set geometry

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- *Date*: Sat, 06 Oct 2007 22:51:28 −0700

On Oct 5, 10:20 pm, David R Tribble <da...@xxxxxxxxxxx> wrote:

> Can I see the point that's halfway between the center
> and the right edge? Is that the point at BigUn/2
> (or maybe BigUn/4, I'm losing track) that's a finite
> distance from the center (because all the points in
> my square are a finite distance from the center)?
> I forget, is BigUn/2 a finite number or not?

As usual, I've tried to find a way, using hyperreals, to make sense of all this Big'Un/Lil'Un/iota, etc.

We begin by identifying Big'Un with a particular hyperreal −− might as well use the equivalence class containing the identity sequence:

B = {1, 2, 3, 4, 5, 6, ...}

(using B for Big'Un, of course). This answers the last question immediately −− Big'Un/2 is still an infinite hyperreal (B/2), as in B/4, as well as B/n for any finite n. Proceeding, if we were now to let Z* denote the set of hyperintegers (i.e., the set of all hyperreals with at least one sequence exclusively of integers in its equivalence class), then we define P, the set of "points," as the set of all hyperreals X such that BX is an element of Z*.

The smallest positive element of P is 1/B. Now 1/B is obviously infinitesimal −− and we'll denote that infinitesimal as L (for Lil'Un). We observe a few things about the set P:

−− For every standard real r, there exists a point X whose shadow (standard part) is r. To construct such a point, we consider the hyperreal:

X = {floor(r), floor(2r)/2, floor(3r)/3, floor(4r)/4, ...}

−− With a gratuitous choice of ultrafilter, one can make
Q (the set of all standard rationals) a subset of P. One
such ultrafilter is one containing the set of factorials:

{1, 2, 6, 24, 120, 720, 5040, 40320, 362880, ...}

It is interesting to see what sort of geometry arises if
we used these "points" instead of the standard reals.

So L is indeed the next point after 0. Of course, there
are still hyperreals between zero and L, but none of
these would be elements of P. I assume this is what
TO means by "first−level," "second−level" infinitesimals.

So we have L/2, L/4, ..., and even L^2 as TO's
"second−level" infinitesimal. Presumably L^3 would be
"third−level" infinitesimal, L^n would be "nth−level"
infinitesimal, and ultimately L^B, which would be a
"Big'Unth−level" infinitesimal.

RF, on the other hand, intends his iota to be nilpotent,
so there are no higher−level infinitesimals.

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