Re: ranges of integer polynomials

Source: http://sci.tech−archive.net/Archive/sci.math/2007−10/msg01355.html

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- *From*: quasi <quasi@xxxxxxxx>

- *Date*: Sun, 07 Oct 2007 12:16:41 −0400

On Sun, 07 Oct 2007 08:06:34 −0700, adler.math@xxxxxxxxx wrote:

> Below is the requested construction of:
>
> A Positive Polynomial whose Range is not Recursive
>
> Let E be a recursively enumerable set of positive integers. For any non−negative n, let g(n) be the n−th prime.
>
> Let P(x, y_1, y_2, \dots) be a "Matijasevic" polynomial for g(E), that is, a polynomial such that x is in g(E) if and only if there exist y_1, y_2, and so on such that P(x, y_1, y_2, \dots) = 0.
>
> Let Q(x_1, x_2, x_3, x_4, y_1, y_2, \dots) be the polynomial
>
> (x_1^2 + x_2^2 + x_3^2 + x_4^2 + 1))(1−3P)^2.
>
> The polynomial Q is positive. If n is in g(E), there are values of the variables such that P=0, and therefore that Q=n.
>
> For all other values of the variables, (1−3P)^2 is divisible by a (varying) square greater than 1, and therefore so is Q.
>
> Suppose now that E is non−recursive. The g(E) is non−recursive. But g(E) is the intersection of the range of Q with the set of primes. Thus the range of Q cannot be recursive.
>
> A Remark on Density
>
> Variants of this idea can be used to deal with density questions. We don't want to map to the primes, a thicker set is useful, like the square−free positive integers. The uninteresting part of the range of Q will be thin, so, if we are working with g(E) of positive density, it will not affect the density of the range of Q.

Looks good.

Some very reusable ideas as well.

Thanks.

Thus, as you suggest, it looks like the density problems can also be resolved by a modified version of the same method.

But let me point out that once again, you replied to the wrong message in the thread, and of course, you also failed to quote the relevant parts of the message you should have replied to. Most newsreaders have an option for automatically quoting the prior message. You can then manually delete the parts that are not relevant. Also, as a minimum, the username of prior poster should show at the top, and if there were nested prior replies (that are relevant), the other usernames should also show, in reverse time order. If you read a few replies by the regular posters, you'll see how it's supposed to look.

In any case, if you refer back to the message that you were supposed to reply to, the one where I asked for your construction, I also gave a solution of my own for problem (8). Mine seems a lot simpler than yours, but is it correct?

quasi
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