Re: RAF: Rational numbers, irrational numbers: each dense in real numbers

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- *From*: "Ross A. Finlayson" <raf@xxxxxxxxxxxxxxx>
- *Date*: Sun, 14 Oct 2007 21:44:36 −0700

On Oct 14, 1:33 pm, rem6...@xxxxxxxxx (Robert Maas, see http://tinyurl.com/uh3t) wrote:

> (RAF is getting as notorious as AP and JSH lately, sigh.)
>
> > From: "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxx>
> > when Goedel claims that no consistent theory can be complete
>
> No, he says no consistent *AND* arithmetic−containing *AND* finitely expressible theory can be complete in the sense of deciding all well−formed sentences.
>
> > and that thus there are "true" statements about the objects of the theory that are not theorems of the theory, I disagree with that, because I think "true" means "provable."
>
> You are wrong: If you can't prove a sentence is true, and you can't prove it's false, then what the fuck, is it true or not? What do you say? Are such sentences true or false? Remember that if S is true then notS is false, so you can't simply make them all true or all false. Or do you look at the first quantifier, if it starts "FOR ALL" then it's true but if it starts "THERE EXISTS" then it's false?
>
> Goedel proved there exists at least one WTF sentence, in fact probably an infinite number of independent WTF sentences. What do you say? How do you assign truth to sentences that can be neither proven nor disproven? You can't just guess, because you'd need an infinite number of guesses to get all of them decided by fiat.
>
> > If Goedel could separate that collection into consistent and inconsistent theorems,

There's no such thing as an inconsistent theorem.

> About omega + 1 > omega, that was a typographical error, excuse me.

That's a lie. There's just no way you could move your fingers to
type omega + 1 and have 1 + omega appear, or vice versa.
It was a *mental* lapse you suffered, a gaffe not a typo.

> I don't ramble incoherently about mathematics,

I disagree: "inconsistent theorem" is incoherent.

> You can't have the simple predicate x=x to define a set in ZFC.

I believe you may have said something correct there.
Is that a strawman you denied there?
I never saw anyone attempt to use x=x to define a set. Have you?

> in theories containing the universe of ZFC, ZFC contains itself.

It depends on what you mean by "contains", whether you mean
subset or element. It makes a difference, an essential difference.
{1, 2, {5}} contains {5} as an element, and {1} as a subset,
but not vice versa.
{1, 2, {1}} contains {1} as both subset and element, but not for
the same reason.
Should I believe you have no idea what the difference is?

> The above defined transfinite recursion schema about sets dense
> in the reals shows ZFC inconsistent.

I'm guessing you're wrong about this claim.
Can you get ahold of a ZFC interactive theorem−proving program
(you enter your premises, and the rules of inference, and it
checks each step is correct)
and try to make it accept your alleged derivation of an inconsistency?
If you succeed, please post the ID of the program you've used and a
transcript of the session. (Post on Web page, with summary and URL here.)

OK, replace "theorem" with sentence or formula, where theorems are consistent formulae. Goedel can't separate consistent and inconsistent statements. If the theory is incomplete there are properties of the objects not accounted for by the axioms, so there is some tacit axiomatization of those properties, ir they wouldn't hold, and the argument that the theory is thus not finitely (or recursively) axiomatized follows as above. If you can't prove a sentence true, and can't prove it false, then there is something about the objects that you don't know, but the objects are totally specified by the axioms. So, then either the axioms are false, i.e., not true representations of the true objects, or there are anonymous axioms, and the theory isn't recursively axiomatized.

About typing omega + 1 = omega when omega + 1 > omega, you'll notice in context that was meant, there was already a reply to Moe about that inconsequential error, where my fingers can reach from > to = on the keyboard quite easily.

Consider Metamath, an automated theorem prover (ATP). It seems that metamath's proof that the reals are uncountable is circular. Plug in rationals. That is to say, in at least one incarnation of Cantor's first proof (of the uncountability of the reals) as implemented in Metamath, Cantor/Megill nested intervals, the rationals were interchangeable with the reals in the development, showing them uncountable.

Browsing Metamath, consider theorem elisseti: http://us.metamath.org/mpegif/elisseti.html (element is set), that a member of a class is a set, with the class of ordinals member of class V. That seems quite controversial, to say that if a class is a member of a class that it's then a set. Maybe instead it's just a faulty description. I track back through the developments, eg http://us.metamath.org/mpegif/elisseti.html , and, it seems to say that, uh, if a class A is an element of a class B then it's a set, using class equality instead of set equality. So, in Metamath, the universal class is the collection of all elements satisfying identity, and, in Metamath, all classes satisfy identity, and, in Metamath, an element of a class is a set, and, each class is an element of the universal class. Thus, in Metamath, each class, for example the class of Ordinals, is a set.

Then, in onprc, it is shown that no set contains all the ordinal numbers. So, then On =/= On, else it would be an element of V and a set, where V is simply defined as the class of all classes, sets, that are equal to themselves. (Identity, of an object being itself, is generally assumed to hold.) As well, it is stated that universal quantification is unrestricted.

http://us.metamath.org/mpegif/con0.html
http://us.metamath.org/mpegif/df−cleq.html

http://us.metamath.org/mpegif/df−v.html

Second, then there is to be described an infinite set with the well−
ordering thus that via transfinite induction/recursion it exists and
is uncountable. Then, the property to show that holds for transfinite
induction is that for a given ordinal, either it is less or equal than
the cardinality of the irrationals and thus there exists uncountably
many irrationals left from which to select, or it is greater than the
cardinality of the irrationals.

http://us.metamath.org/mpegif/tfi.html

That is where the desired property for a given ordinal (that there are
more elements in the interval (0, p_alpha) for ordinal alpha less than
the cardinality of the irrationals) holds for ordinals less than or
equal to the cardinality of the irrationals, where for higher ordinals
the property would not consistently hold, but it is not necessary that
it does. So, in the course of values over all ordinals, for ordinals
less than or equal to the cardinality of the irrationals there are at
least that many remaining in the interval (p_alpha, 0). (Otherwise,
there wouldn't be that many in the interval.) For ordinals greater
than the cardinality of the irrationals, they as well satisfy the
property in being greater than the cardinality of the irrationals.
Then, there are as many elements p_alpha as there are ordinals alpha
that are less than or equal to the cardinality of the irrationals.
Then, that holds for sufficiently many irrationals, for each of which
can be displayed a distinct rational, that theorem contradicts another
in the theory.

Basically for each partition of the irrationals intersecting the
interval (p_alpha, 0) into (p_alpha, p_alpha+) and (p_alpha+, 0), each
partition has the same cardinality.

The rationals and irrationals are each dense in the reals.

So, ZFC is inconsistent.

Ross

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