Re: What is the meaning of the expression E^F, where both E and F are sets?

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- *Date*: Tue, 16 Oct 2007 19:02:57 −0700

On Oct 16, 12:19 pm, "porky_pig...@xxxxxxxxxxx" <porky_pig...@my−deja.com> wrote:

> In both Halmos' Naive Set Theory and Suppes' Axiomatic Set Theory, the expression E^F is introduced without any comments. The same text by Suppes (if you skip a few chapters) refers to E^F as a set of all functions f: E −> F.

In the canon of set theory, 0 = {}, 1 = {0}, 2 = {0,1}, 3 = {0,1,2}, etc. so that A^2 would be the set { (a0,a1): a0, a1 in A } which is equivalent to the set of functions from 2 to A, noting the correspondence a(0) = a0, a(1) = a1. Similarly, A^3 consists of ordered triples.

This generalizes. If, instead of numeric parameters, the functions have non−numeric parameters, you still want to say that the members of A^X are ordered X−tuples. If the size of the set X is the number n, then this is equivalent to A^n, but more direct.

Therefore, the exponential A^X consists of all ordered X−tuples drawn from A which is equivalent to the set of all functions mapping X to A.

An alternate notation is (X−>A). The laws of exponents A^{X x Y} = A^X x A^Y establishes an equivalence between the respective sets. In the alternate notation this becomes (X x Y) −> A = (X −> A) x (Y −> A). This is also a logical tautology, if ()x() is interpreted as conjunction and ()−>() as implication. For the empty set, you have A^0 = 1 which is rendered as (0−>A) = 1. This, too, is a tautology if 0 is interpreted as "false" and 1 as "true".

For compound exponents, you have the equivalence (A^X)^Y = A^(Y x X}, which becomes (Y−>(X−>A)) = (Y x X) −> A. This, too, is a tautology in logic, when interpreted as a logical statement.

Thus, there is a deep−seated link between the if−then connective in logic and the exponential operator.

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