Re: smallest positive integer that has exactly k divisors

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Phil Carmody wrote:

> "mensanator@xxxxxxxxxxx" <mensanator@xxxxxxx> writes:
>
> > > > ## Did you really mean 2000 factors? You do know that the
> > > > ## number of divisors is 2**factors, don't you?
> >
> > > Really? Then how can 12 have 6 divisors?
>
> > It doesn't, it has 6 UNIQUE divisors.

I haven't been following this thread, so if this answer has been given before, please forgive the redundancy.
Assume the OP is interested in the number of unique divisors of n. That is the product of the exponents, each incremented by 1, in the prime factorization of n (counting 1 and n as divisors). So, given k, the desired number of factors of n, first factor k into primes. Say there are x (not necessarily unique) prime factors of k. The desired n is the product of the smallest x prime numbers, in increasing order, raised to exponents which are one less than the prime factors of k, in decreasing order.
So, what is the smallest integer with 18 divisors? k=18=3*3*2, so n=(2**(3−1))*(3**(3−1))*(5**(2−1))=180, with divisors 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90, 180.
If you don't want to count 1 and/or n as divisors, factor a value of k that is 1 or 2 more than the number of divisors you want.

−−Mike Amling
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