

## Re: ? dual spaces

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"Jack Schmidt" <[Jack.Schmidt.SciMath@xxxxxxxxxx](mailto:Jack.Schmidt.SciMath@xxxxxxxxxx)> wrote in message <news:7577067.1184781337314.JavaMail.jakarta@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>

I don't think there is much hope in general, but for some specific types of nonlinear functions you may get what you want.

For instance, work with the nonlinear function  $f(x) = xAx^t$ , where  $A$  is an  $n \times n$  matrix,  $x$  is a  $1 \times n$  row vector, and  $f(x)$  is a  $1 \times 1$  scalar. You can replace  $A$  by  $(A+A^t)/2$  without changing the function, and this matrix is symmetric, and so normal. Write  $(A+A^t)/2 = UDU^t$  with  $U$  unitary and  $D$  diagonal, then  $f(xU^t) = \text{Sum}(d_i x_i^2, i=1..n)$ . In some sense then, you might be interested in those  $i$  with  $d_i=0$ , thinking of it like the kernel.

This reminds me something. If we define  $f(x) = x^*Ax/x^*x$ , where  $A$  is Hermitian and  $x$  is  $N$ -by-1. Then  $\lambda_{\min} \leq f(x) \leq \lambda_{\max}$ , where  $\lambda$  denotes eigenvalue of  $A$ .

Can one say that any  $x$  in  $A$ 's domain can be expressed by  $A$ 's  $e$ -vectors or one cannot because this is a nonlinear map?

But one can still form a smaller subset by constraining  $x_{\text{con}}$  to be the subset "spanned" (if one may write that...) by some of the  $e$ -vectors of  $A$ . More specifically, let  $u_n$  denote  $e$ -vectors corresponding to  $e$ -values sorted from largest to smallest  $e$ -value  $\lambda_n$ . If  $x_{\text{con}} = \{u_3, u_5\}$  and  $\dim(A) = 7$ , will the image

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be limited to  $\lambda_5 \leq f(x_{\text{con}}) \leq \lambda_3$ ?

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