

# Re: Implementable Set Theory and Consistency of ZFC

---

*Source:* <http://sci.tech-archive.net/Archive/sci.math/2007-10/msg04560.html>

---

- *From:* David C. Ullrich <ullrich@xxxxxxxxxxxxxxxxxxxx>
  - *Date:* Thu, 25 Oct 2007 06:17:37 -0500
- 

On Wed, 24 Oct 2007 15:57:41 +0200, Han de Bruijn  
<Han.deBruijn@xxxxxxxxxxxxxxxx> wrote:

David C. Ullrich wrote:

On Wed, 24 Oct 2007 09:50:42 +0200, Han de Bruijn  
<Han.deBruijn@xxxxxxxxxxxxxxxx> wrote:

MoeBlee wrote:

On Oct 23, 4:23 am, Han de Bruijn  
<Han.deBru...@xxxxxxxxxxxxxxxx> wrote:

1. Extensionality
5. Specification X. Infinity
2. Empty set
6. Substitution
3. Pairing
7. Power Set
4. Union
8. Foundation
9. Choice

And, as I've said, in this "model", only (1-4) are necessary as axioms, because (5-9) appear as theorems. And (X) is not part of the "model".

I just saw Virgil's post, which made me realize I overlooked that you said 5-9 are theorems of 1-4!

You're wrong. None of 5-9 is derivable

Re: Implementable Set Theory and Consistency of ZFC

from all of 1–4.

What you might mean is that in a certain model of 1–4, we have that 5–9 are true in that model. But that does not ENTAIL that any of 5–9 are theorems of 1–4.

You are wrong. Read the article.

Just for fun I downloaded it and looked at it.  
The supposed proof of (5) is this:

Think you mean (8) Foundation, throughout the following.

"Theorem. The minimal element in a set, when implemented as a natural array, is always disjoint from the set.  
Proof. This is a direct consequence of the above lemma: the leftmost bit position of an integer  $\geq 0$  has a numeric value which is always less than the numeric value of the integer itself.  
This completes the proof, of the assertion that the axiom of Foundation is just a Theorem in our implementable set theory."

All that stuff about bit positions is specific to your "model". Sure enough, exactly as Jesse conjectured, it's not a proof that 5 follows from 1–4 as you insist, it's just a proof that 5 is true in your "model".

At most it's a proof that 5 is true of the hereditarily finite sets. That's not what you've been insisting it is, in particular it is simply not a proof that 5 follows from 1–4.

Any (implementable) set is a hereditarily finite set, anyway.

Which of course is not surprising, since 5 does not follow from 1–4.

Once you've entered my universe, you're plain wrong.

Erm, that's silly. As people keep saying over and over, if you're using words to mean something other than what they usually mean (for example if words have different meanings in "your universe") you need to explain what those meanings are. Or better, use different words.

In fact 5–8 do not follow from 1–4. The explanation for the problems with your "proof" that they do is exactly as people have been saying – showing that 5–8 are true in a model that happens to satisfy 1–4 does not prove that they follow, for that you need to show that 5–8 hold in every model of 1–4. Many people explain this over and over – instead of trying to understand their perfectly correct corrections you complain about the fact that they haven't read your "proof" because they're lazy. This is just silly.

But I think much of the confusion rather stems from poor terminology.

[http://en.wikipedia.org/wiki/Computable\\_function](http://en.wikipedia.org/wiki/Computable_function)

Modified quote. Replacing "function" by "SET":

According to the Church–Turing thesis, computable SETs are exactly the SETs that can be calculated using a mechanical calculation device given unlimited amounts of time and storage space. Equivalently, this thesis states that any SET which has an algorithm is computable. Consequently: "Implementable Set Theory" has been renamed to "Computable Set Theory".

[http://hdebruijn.soo.dto.tudelft.nl/jaar2007/set\\_theory.pdf](http://hdebruijn.soo.dto.tudelft.nl/jaar2007/set_theory.pdf)

Does that make things better? Is it an improvement?

Huh? It has no bearing on the question of whether 5–8 follow from 1–4. They don't.

Han de Bruijn

\*\*\*\*\*

David C. Ullrich

