Re: Funky random number choosing

Source: http://sci.tech−archive.net/Archive/sci.math/2007−10/msg04652.html

- *From*: Michael Press <rubrum@xxxxxxxxxxx>
- *Date*: Thu, 25 Oct 2007 11:17:00 −0700

In article
<1193154890.814743.172740@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>
, thomasjack <thomasjack@xxxxxxxxx> wrote:

> Let {r_n} be a sequence of reals in (0,1] generated by a uniform random number generator, with n=0,1,2... Let x be the smallest r_n for which r_(n+1) > r_n. What's the expected value of x?
>
> For example, if the sequence of random numbers came up "0.06, 0.21, 0.95, 0.93...", then x would be 0.95.

I think you mean the first r_n for which r_{n+1} < r_n.

> After a few million trials I believe the expected value is around 0.718 (could it be e−2?), but I am having trouble deriving the exact value.

m uniformly distributed values can be ordered in m! ways.
The number of permuations where r1 < r2 < ...< r_{m−1} > r_m is m−1.
The probability is (m−1)/m!.
The expected gap for m uniformly distributed values is 1/(m+1).
The expected value of r_{m−1} is

sum_{2 <= m}(1 − 1/(m+1)).(m−1)/m!
= sum_{2 <= m}[m/m! − 1/m! − (1/(m+1).(m+1 − 2)/m!]
= sum_{2 <= m}[m/m! − 1/m! − 1/m! + 2/(m+1)!]
= (e−1) − 2(e−2) + 2(e − 2 − 1/2)
= e − 1 − 1
= e − 2.

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