Re: .99999....=1

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- *From*: Proginoskes <CCHeckman@xxxxxxxxx>
- *Date*: Sun, 28 Oct 2007 09:58:30 −0000

On Oct 27, 9:00 pm, lwal...@xxxxxxxxx wrote:

> On Oct 26, 11:58 pm, Proginoskes <CCHeck...@xxxxxxxxx> wrote:
>
>> On Oct 26, 3:34 pm, lwal...@xxxxxxxxx wrote:
>>
>>> 0.999... = 1 − 10^N.
>>
>> I think you mean 1 − (0.1)^N here.
>
> Of course I did. 1 − 10^N makes no sense here. Naturally,
> you are right here.
>
>> Since (0.1)^N is an infinitesimal,
>> and due to the Transfer Principle, this equation implies that
>>
>> 0.999... = 1.

Well, in R at least.

> As far as the hyperreals are concerned, there are some who
> believe that a decimal such as 0.abcd... represents the
> following hyperreal
>
> {0.a, 0.ab, 0.abc, 0.abcd, ...}
>
> in the sense that _every_ sequence of reals −− regardless of
> whether the sequence even converges −− corresponds to a
> unique hyperreal. In the case where the sequence does
> converge, as in this case, the standard part of the hyperreal
> is in fact that limit. (This is mentioned in the Keisler link
> you provided above.) But in this example, the infinitesimal

part is never zero unless only finitely many of the digits are nonzero. By this approach, 0.999... is 1 − (0.1)^N, just as 0.333... is exactly one−third of this, (1 − (0.1)^N) / 3.

The other approach is to consider that 0.999.. already has a meaning as a standard real, namely 1, since the Cauchy sequence converges to unity. And since the hyperreals are an extension of the reals, every real, including 0.999..., must be a hyperreal and must equal the same real. So by this approach, 0.999... = 1, and any number with a decimal expansion must equal its own standard part. Hyperreals such as 1 − (0.1)^N cannot be represented as a decimal.

The Wikipedia article to which I linked in another post also mentions Lightstone's extended decimal representation of the hyperreals:

"In his formalism, there are two natural extensions of 0.333..., neither of which falls short of 1/3 by an infinitesimal:

0.333...;...000... does not exist, while
0.333...;...333... = 1/3 exactly."

As we see here, Lightstone is allowing the decimal places to be indexed, not by the standard naturals (as with the standard reals), but by the hypernaturals (the analogue of the naturals in the hyperreals). We see that if every digit, whether in a standard natural position or in a hypernatural position, is three, then the number is 1/3 (and thus the number 0.999...;...999... = 1 exactly).

One may wonder why 0.333...;...000... does not exist. This reminds me of Internal Set Theory, the version of set theory which allows for nonstandard hyperreals. In IST, N actually _is_ the set of hyperreals. The set of standard naturals can not exist in IST −− it is an example of what is known as "illegal set formation." So with 0.333...;...000..., the set of all hypernaturals whose digit is three would be the set of all standard natural numbers −− which is an instance of illegal set formation. So 0.333...;...000... does not exist.

Of course, if 1/3 = 0.333...;...333... in Lightstone's notation, what about 1/11 = 0.090909...? Or even worse 1/7, which would be 0.142857142857...? The answer is that it would depend on the choice of ultrafilter. And of course, trying to write sqrt(2) or pi in Lightstone's notation would be an impossible task. (Such a notation would still _exist_, but one would not be able to calculate it.)

Notice that 0.999...;...999... notation is reminiscent of some of Tony Orlow's notation for his infinitesimals. (See the

> Cantor threads for more info on T−riffic numbers.) This also comes up in the Archimedes Plutonium threads, where it is sometimes asked whether the digits of the AP−adic numbers (a generalization of p−adics) are themselves indexed by AP−adics. (Once again, please refer to the relevant AP threads for more info.)

Don't remind me about AP! The past few nights I haven't felt like even reading what he has to say, and it might be better for me if I stayed that way. 8−)

BTW, I was one of the people who pointed out that the decimal places of the AP−adics cannot be indexed by AP−adics, as AP believes there is only one infinity.

−−− Christopher Heckman

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