

# Re: Implementable Set Theory and Consistency of ZFC

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Jesse F. Hughes wrote:

Han de Bruijn <Han.deBruijn@xxxxxxxxxxxxxxxx> writes:

So, even if I don't make use of (5-8), a proof of A from (1-4) is a proof from (1-8) ?

Of course.

So, even if I say "there exists a Foo", then such a statement is a valid premise for proving that the integral of  $1/t$  from 1 to  $x$  is  $\ln(x)$  ? Weird ..

The statement can be proved in the theory consisting of the usual axioms for real analysis and "there exists a Foo", yes. Do you think that every theorem of ZFC uses every axiom of ZFC in its proof?

It seems we have a different picture in our mind about the meaning of an implication  $A \Rightarrow B$ , as has been pinpointed by Ullrich as well. This is what I say about in in my article:

Another philosophical note is in place, when we are saying that we "make with an axiom" and denote this as an implication  $A \Rightarrow B$ . In common mathematics, the implication  $\Rightarrow$  just means what is defined by a truth table in propositional logic. But there is another form of mathematics, called constructivism. Within constructivist mathematics, an implication has a more "operational" meaning, like: given A, we can construct B from A. So if we say "make with an axiom", then it is expressed herewith that we adhere to the constructivist meaning of an implication. End of philosophical note.

## Re: Implementable Set Theory and Consistency of ZFC

I think that you and Ullrich adhere to the common "material implication" of mathematical logic, where there is no place for axioms that "cause" a theorem (so to speak). In the latter sense there is no room for premises like "there exists a Foo". The axiom of Infinity is of the latter kind.

Han de Bruijn

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