

# Re: Implementable Set Theory and Consistency of ZFC

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- *From:* MoeBlee <jazzmobe@xxxxxxxxxxxx>
  - *Date:* Wed, 31 Oct 2007 11:45:10 -0700
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On Oct 31, 6:12 am, Han de Bruijn <Han.deBru...@xxxxxxxxxxxx> wrote:

Jesse F. Hughes wrote:

Han de Bruijn <Han.deBru...@xxxxxxxxxxxx> writes:

So, even if I don't make use of (5-8), a proof of A from (1-4) is a proof from (1-8) ?

Of course.

So, even if I say "there exists a Foo", then such a statement is a valid premise for proving that the integral of  $1/t$  from 1 to  $x$  is  $\ln(x)$  ? Weird ..

The statement can be proved in the theory consisting of the usual axioms for real analysis and "there exists a Foo", yes. Do you think that every theorem of ZFC uses every axiom of ZFC in its proof?

It seems we have a different picture in our mind about the meaning of an implication  $A \Rightarrow B$ , as has been pinpointed by Ullrich as well. This is what I say about in in my article:

Another philosophical note is in place, when we are saying that we "make with

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an axiom" and denote this as an implication  $A \Rightarrow B$ . In common mathematics, the implication  $\Rightarrow$  just means what is defined by a truth table in propositional logic. But there is another form of mathematics, called constructivism. Within constructivist mathematics, an implication has a more "operational" meaning, like: given A, we can construct B from A. So if we say "make with an axiom", then it is expressed herewith that we adhere to the constructivist meaning of an implication. End of philosophical note.

I think that you and Ullrich adhere to the common "material implication" of mathematical logic, where there is no place for axioms that "cause" a theorem (so to speak). In the latter sense there is no room for premises like "there exists a Foo". The axiom of Infinity is of the latter kind.

Then please specify an exact constructivist logic. Because, for example, I am not aware that intuitionistic logic (even with its semantics for ' $\rightarrow$ ') contradicts monotonicity of deduction (if someone informs me that intuitionistic logic does contradict monotonicity of deduction, then I'll look into that). So, if your logic is not intuitionistic, please specify your system of logic.

MoeBlee