Re: Implementable Set Theory and Consistency of ZFC

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* *From*: Virgil <virgil@xxxxxxxxxxx>
* *Date*: Wed, 31 Oct 2007 16:24:49 −0600

In article <b024b$47287f47$82a1e228$29885@xxxxxxxxxxxxxxxx>,
Han de Bruijn <Han.deBruijn@xxxxxxxxxxxxxx> wrote:

> Jesse F. Hughes wrote:
>
>> Han de Bruijn <Han.deBruijn@xxxxxxxxxxxxxx> writes:
>>
>>> So, even if I don't make use of (5−8), a proof of A from
>>> (1−4) is a
>>> proof from (1−8) ?
>>
>> Of course.
>>
>>> So, even if I say "there exists a Foo", then such a statement is
>>> a
>>> valid premise for proving that the integral of 1/t from 1 to x
>>> is
>>> ln(x) ? Weird ..
>>
>> The statement can be proved in the theory consisting of the usual
>> axioms for real analysis and "there exists a Foo", yes. Do you think
>> that every theorem of ZFC uses every axiom of ZFC in its proof?
>
> It seens we have a different picture in our mind about the meaning of an
> implication A => B , as has been pinpointed by Ullrich as well. This is
> what I say about in in my article:
>
>> Another philosophical note is in place, when we are saying that we "make
>> with
>> an axiom" and denote this as an implication A => B. In common

> > mathematics,
> > the implication => just means what is de ned by a truth table in
> > propositional
> > logic. But there is another form of mathematics, called constructivism.
> > Within
> > constructivist mathematics, an implication has a more "operational"
> > meaning,
> > like: given A, we can construct B from A. So if we say "make with an
> > axiom",
> > then it is expressed herewith that we adhere to the constructivist meaning
> > of
> > an implication. End of philosophical note.

> I think that you and Ullrich adhere to the common "material implication"
> of mathematical logic, where there is no place for axioms that "cause" a
> theorem (so to speak).

> In the latter sense there is no room for premises
> like "there exists a Foo". The axiom of Infinity is of the latter kind.

When one speaks of an axiom system in HdB speak then,only those
statements which REQUIRE all of the axioms can be called theorems. This
is not the usual understanding, even among constructionists.

Even among the most rigorous of constructionists, this is not at all
what an axiom SYSTEM is held to be. Within an axiom system, even the
most extreme of constructionists still allow production of theorems not
requiring every axiom of the system in their construction.

It is hard to see why anyone would want it otherwise, but HdB seems to
have reasons that reason knows no of.
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