Re: Algebra with isomorphism and group.

Source: http://sci.tech−archive.net/Archive/sci.math/2007−11/msg03015.html

---

- *From*: Derek Holt <mareg@xxxxxxxxxxxxx>
- *Date*: Thu, 15 Nov 2007 06:13:31 −0800 (PST)

---

mina_world wrote:

> "mina_world" <mina_world@xxxxxxxxxxx> wrote in message
> news:fhhasu$q87$1@xxxxxxxxxxxxxxxxxxx
>
> > Hello sir~
> >
> > G is a group.
> >
> > f : G −> f[G] is isomorphism.
> >
> > then,
> > Can I say that f[G] is group unconditionally ?
>
> In fact, my question derive from proof of Cayley's theorem.
> Namely,
> Every group is isomorphic to a group of permutations.
> pf)
> Let G be a group.
> Let f : G −> S_G by f(x) = g_x for all x in G.
> Let g_x : G −> G by g_x(g) = xg for all g in G.
>
> so, g_x is a permutation of G. (easy to show)
>
> Since f is homomorphic and 1−1 and onto,
> G ~ f(G) (isomorphic)
>
> Can I say that f(G) is (sub)group automatically ?

Yes, in general if G and H are groups and f: G −> H is a homomorphism,
then the image f(G) of f is a subgroup of H. The proof is
straightforward.

(Also, the kernel K of f is a normal subgroup of G and G/K is
isomorphic to f(G).)

Derek Holt.
.