

Re: The infinitely small number b

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- *From:* Venkat Reddy <vreddyp@xxxxxxxxxx>
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On Nov 18, 3:44 am, lwal...@xxxxxxxxxx wrote:

On Nov 16, 7:21 pm, Venkat Reddy <vred...@xxxxxxxxxx> wrote:

b is not equal to b in arithmetic sense (addition, subtraction).

I enjoy thinking about these alternate number systems and how to make them more rigorous, but there can be no generalization of the real numbers in which we do not even have $b = b$.

Otherwise, these numbers remind me a bit more of Conway's surreal numbers than Robinson's hyperreals --- in particular, the surreals whose birthday is on or before the first infinite day. For those of you who are already familiar with the surreals, recall that these numbers include:

- * All the standard real numbers
- * an infinite number (Conway's "omega," venkat's "inf"),
- * its additive inverse (Conway's "negative omega," venkat doesn't mention negative numbers)
- * an infinitesimal number (Conway's "epsilon," venkat's "b" of course)
- * numbers which differ from a dyadic rational by this infinitesimal (venkat's "n + b," "n - b" above)

Some of venkat's rules work for these surreals:

$$\begin{aligned}b + 0 &= b \\b - 0 &= b \\b * 0 &= 0 \\(b / 0) &\text{ is undefined and not to be taken as inf.} \\b / b &= 1\end{aligned}$$

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Many of the venkat rules disagree with the surreals —
and their correct values in the surreals are numbers
whose birthday is beyond the first infinite birthday:

$$b + b = 2b$$

$$b * b = b^2$$

$$b - b = 0$$

$$n * b = nb$$

$$1 / 0 = \text{inf}$$

$$n / 0 = \text{inf} * n$$

I'm not quite sure why VR defines b / b as 1, but
leaves $b - b$ undefined

Thanks for noticing. That's an error and I've posted a correction
immediately. b/b is undefined too.

– venkat

— or in general, argues that
 b works differently "arithmetically" (addition and
subtraction) as "geometrically" (multiplication).