

Re: The infinitely small number b

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- *From:* Venkat Reddy <vreddyp@xxxxxxxxxx>
 - *Date:* Sun, 18 Nov 2007 04:18:56 -0800 (PST)
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On Nov 18, 9:56 am, mike3 <mike4...@xxxxxxxxxx> wrote:

On Nov 17, 7:33 pm, Venkat Reddy <vred...@xxxxxxxxxx> wrote:

On Nov 18, 3:44 am, lwal...@xxxxxxxxxx wrote:

On Nov 16, 7:21 pm, Venkat Reddy <vred...@xxxxxxxxxx> wrote:

b is not equal to b in arithmetic sense
(addition, subtraction).

I enjoy thinking about these alternate number systems and how to make them more rigorous, but there can be no generalization of the real numbers in which we do not even have $b = b$.

Otherwise, these numbers remind me a bit more of Conway's surreal numbers than Robinson's hyperreals -- in particular, the surreals whose birthday is on or before the first infinite day. For those of you who are already familiar with the surreals, recall that these numbers include:

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- * All the standard real numbers
- * an infinite number (Conway's "omega," venkat's "inf"),
- * its additive inverse (Conway's "negative omega," venkat doesn't mention negative numbers)
- * an infinitesimal number (Conway's "epsilon," venkat's "b" of course)
- * numbers which differ from a dyadic rational by this infinitesimal (venkat's " $n + b$," " $n - b$ " above)

Some of venkat's rules work for these surreals:

$$\begin{aligned}b + 0 &= b \\b - 0 &= b \\b * 0 &= 0 \\(b / 0) &\text{ is undefined and not to be taken as} \\&\text{inf.} \\b / b &= 1\end{aligned}$$

Many of the venkat rules disagree with the surreals --- and their correct values in the surreals are numbers whose birthday is beyond the first infinite birthday:

$$\begin{aligned}b + b &= 2b \\b * b &= b^2 \\b - b &= 0 \\n * b &= nb \\1 / 0 &= \text{inf} \\n / 0 &= \text{inf} * n\end{aligned}$$

I'm not quite sure why VR defines b / b as 1, but leaves $b - b$ undefined

Thanks for noticing. That's an error and I've posted a correction immediately. b/b is undefined too.

– venkat

Re: The infinitely small number ϵ

Why do you need $\epsilon - \epsilon$ to "be" (heh) undefined?
Why not $\epsilon - \epsilon$ equal zero?

It is not equal to zero because of the first rule: ϵ is not equal to itself. ϵ is not a single number but the "smallest" number. So it is a quality of the things (quoting Buddha Thucydides referring Gauss).

Also, why don't you like the surreal numbers? They do what you want, make infinitesimals. And they make lots more than your ϵ -number (or whatever that other silly name was you gave it), and the best thing is they've got a rigorous definition, which yours doesn't.

Yes, I'm not formally trained in mathematics, so please pardon the lack of rigor in my writings. Regarding surreals, I have just now taken a look at them on wikipedia. Conway's infinitesimal allows $\epsilon = \epsilon$ and in general the operations go too far making the infinitesimal look like a single definite number, but not a symmetric counterpart of infinity in geometric sense (or multiplicative sense). So I think ϵ does not adapt to the definition and properties of surreal infinitesimal. I haven't looked at hyperreals – will take a look and see if ϵ is superfluous.

– venkat

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