Re: Another naive question − limts are meant for not crossing or not reaching?

Source: http://sci.tech−archive.net/Archive/sci.math/2007−11/msg05108.html

---

- *From*: William Hughes <wpihughes@xxxxxxxxxxx>
- *Date*: Mon, 26 Nov 2007 07:46:25 −0800 (PST)

On Nov 26, 9:02 am, Venkat Reddy <vred...@xxxxxxxxx> wrote:

> On Nov 26, 4:57 pm, William Hughes <wpihug...@xxxxxxxxxxx> wrote:
>
> > On Nov 26, 4:29 am, Venkat Reddy <vred...@xxxxxxxxx> wrote:
> >
> > > On Nov 25, 6:53 pm, David W. Cantrell
> > > <DWCantr...@xxxxxxxxxxx> wrote:
> > >
> > > > Venkat Reddy <vred...@xxxxxxxxx> wrote:
> > > >
> > > > > On Nov 25, 10:29 am, quasi
> > > > > <qu...@xxxxxxxx> wrote:
> > > > >
> > > > > > On Sat, 24 Nov 2007 21:22:11 −0800 (PST), Venkat Reddy <vred...@xxxxxxxxx> wrote:
> > > > > >
> > > > > > > On Nov 25, 9:03 am,

William Hughes <wpihug...@xxxxxxxxxxx> wrote:

> On Nov 24, 10:37 pm, Venkat Reddy <vred...@xxxxxxxxx> wrote:
>
> > What do we mean by lim x−>oo 1/2^x = 0 ? That It never crosses zero or it never reaches zero?
> >
> > Fo me, if it can reach then it

can cross as well, for there is no specific change at that point which prevents crossing. This indicates it never reaches zero. So the equality must be wrong.

The sequence 1/2,1/4,1/8... never reaches zero. Desite this fact, the equality is not wrong. The lim x−>oo 1/2^x does not mean "the value that the sequence 1/2^x reaches" it is a real number L with the property that the sequence 1/2,1/4,1/8,... gets and stays arbitrarily close to L (i.e. the difference between L and the series can be shown to be less than any number greater than zero). It is not hard to show that the sequence gets and stays arbitrarily close to 0, and that the sequence does not get and stay arbitrarily close to any other real number. The conclusion is that L=0.

Ok, the equality is not conclusive about x but about some other number called

L to which x is getting and staying arbitrarily close.

Right.

There's hope for you yet.

Then, if the "limit" has nothng to say about whether x reaches zero or not, I guess we both agree that x never reaches zero. Is that right?

In this case,
yes.

But as has been pointed out, even if a sequence (some other sequence) does "hit zero" or "crosses zero", the limit would still be zero, as long as the sequence gets and stays close to zero −− arbitrarily close.

Got it. Some functions could be taking values on both side of zero,
but converging on zero as a limt. I think I've got the answer in my
case − it never hits zero.
Thanks.

Whether 1/2^x actually "hits zero" or not depends on what is used for the domain of the function. But let's look at another example first:

Let f(x) = Sqrt(1 − x^2) with domain (−1, 1).
Then the limit of f(x) as x

approaches 1 from the left is 0, but f never hits 0 since it is strictly positive on its domain. But instead, if we were to use the closed interval [−1, 1] for the domain of f, then f would hit zero when x = 1.

Now for your function g(x) = 1/2^x, the situation is similar. If we take its domain to be just the real numbers, (−oo, +oo), then the limit of g(x) as x approaches +oo is 0, but g never hits 0 since it is strictly positive on its domain. But instead, if we were to use the extended reals, [−oo, +oo], for the domain of g, then g would hit zero when x = +oo, that is, g(+oo) = 0.

That's much clearer. x hits zero only if I include +oo in the domain.

I.e. since +oo is not in the domain, g(x) is not 0 for any x in it's domain.

Now, I'm trying to validate the meaning of the domain (−oo, +oo) that includes "arbitrarily large number", but excludes +/− oo. Isn't this a self contradiction for the meaning of +/− oo?

No. To say that you can choose x to be arbitrarily large does not mean "there exists an x such that for every y, x>y" but "for every y there exists an x(y) such that x(y) > y" (note if y changes x(y) may change). The statements are very different.

What is it really excluding? Some absolutely largest number?

> > Yes, (−oo,oo) is another way of saying each and every real number.
> > There is no "absolutely largest number" in the reals, so
> > means that no real number is excluded.
>
> Doesn't "each and every real number" include infinity? Sorry, I'm just
> asking to know it.
>
> − venkat

No, infinity is not a real number. There are a number of ways we can
add something
called "infinity" to the real numbers, thus getting a system of
numbers that
includes infinity. However, in no case will this system of numbers be
the real
numbers. So although there we can say "infinity is a number" (if we
define
both infinity and number correctly) there is no way to say "infinity
is a real
number".

− William Hughes
.