

Re: Positive/Negative after taking the square root

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Joe wrote :

I'm not totally mathematically inept, but I find

I'm

getting confused
by the following:

I'm confused as to what happens to the +/- in the
following examples:

As a simple example, integrate x^2

$\int \{x^2 dx\}$

Let $x^2 = u$, then $dx = du/(2x)$ and $du = 2x dx =$

$+/-$

$2\sqrt{u}$

Then the integral becomes
 $= \int \{u/(+/-)2\sqrt{u} du\}$
 $= +/- \int \{1/2\sqrt{u} du\}$
 $= +/- 1/3u^{1.5}$
 $= +/- 1/3x^3$ (substitute back in for u)

This is obviously not correct, and the correct

answer

is in taking
only the positive square root when representing dx
in terms of u and
du. However, is there a mathematical/logical

reason

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for discounting
the negative root? After all, if $y^2 = x$, then $y =$
 $\pm\sqrt{x}$

A related problem I encountered when deriving the
derivative of
arcsin:

$$y = \arcsin x$$
$$x = \sin y$$

I have $y' = 1/\cos y$, and wish to write y in terms of

x

$$\text{Now, } \sin^2 y + \cos^2 y = 1$$

$$\sin y = x$$

$$\text{thus } x^2 + \cos^2 y = 1$$

$$\cos^2 y = 1 - x^2$$

$$\cos y = \pm \sqrt{1 - x^2}$$

Thus

$$y' = \pm 1/\sqrt{1 - x^2}$$

Obviously, again this is incorrect. The correct
answer is obtained by
discounting the negative root – again, what is the
mathematical/
logical reason for this?

There are two common mistakes made here that lead to
the confusion:

One: The expression $u^2 = v$ does NOT imply $u = \pm\sqrt{v}$. This abuse of notation doesn't really even
mean anything mathematically, and it only serves to
create ambiguity. What it DOES imply is $|u| = \sqrt{v}$
or equivalently $\text{sgn}(u) \cdot u = \sqrt{v}$.

agreed.

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Two: You cannot make a substitution u for x which has $u' = 0$ for some x ; in general if you make such a substitution the result is not valid. This subtle condition may or may not be covered in your calculus class, but the reasoning behind it will be clear when you take analysis.

can you give examples of that ?

is this about avoiding integral from 0 to 0 ?

i assume here you mean for the given example ...

but this is no longer true for single-valued right ?

also ; can this be used to prove or disprove there are zero's on a line or strip ? (RH :p)

To resolve problem 1 take the proper expression $\text{sgn}(x) \cdot x = \sqrt{u}$ and perform the substitution, then we have formally $dx = 2 \cdot \sqrt{u} / \text{sgn}(x) \cdot du$.

this indeed works.

To

resolve problem 2, we need to demand that $x \neq 0$ on the interval of integration $[a,b]$, i.e. a and b are of the same sign. Thus the integral becomes:

$$\begin{aligned} & \int_{[a,b]} \{x^2 dx\} \\ &= \int_{[a^2,b^2]} \{ \text{sgn}(x) \cdot u / (2 \cdot \sqrt{u}) du \} \\ &= \int_{[a^2,b^2]} \{ \text{sgn}(x) \cdot u / (2 \cdot \sqrt{u}) du \} \\ &= \text{sgn}(x) \cdot \int_{[a^2,b^2]} \{ u / (2 \cdot \sqrt{u}) du \} \\ &= \text{sgn}(x) \cdot 1/3 \cdot \sqrt{u^3} \\ &= \text{sgn}(x) \cdot 1/3 \cdot \sqrt{b^6} - \text{sgn}(x) \cdot 1/3 \cdot \sqrt{a^6} \\ &= \text{sgn}(x) \cdot 1/3 \cdot (|b^3| - |a^3|) \\ &= 1/3 \cdot (b^3 - a^3) \end{aligned}$$

which is the correct evaluation as expected.

regards
tommy1729

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