

Re: Positive/Negative after taking the square root

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On Nov 28, 7:36 am, Taras_96 <taras...@xxxxxxxxxx> wrote:

I'm not totally mathematically inept, but I find I'm getting confused by the following:

I'm confused as to what happens to the +/- in the following examples:

As a simple example, integrate x^2

$$\int \{x^2 dx\}$$

$$\text{Let } x^2 = u, \text{ then } dx = du/(2x) \text{ and } du = 2x, dx = +/- 2\sqrt{u}$$

$$\begin{aligned} \text{Then the integral becomes} \\ &= \int \{u/(+/-)2\sqrt{u} du\} \\ &= +/- \int \{1/2\sqrt{u} du\} \\ &= +/- 1/3u^{1.5} \\ &= +/- 1/3x^3 \text{ (substitute back in for } u) \end{aligned}$$

This is obviously not correct, and the correct answer is in taking only the positive square root when representing dx in terms of u and du. However, is there a mathematical/logical reason for discounting the negative root? After all, if $y^2 = x$, then $y = +/-\sqrt{x}$

When you substitute $u = x^2$ you're losing information about which of the two possible values of x you're dealing with. (This always happens when the inverse function is multi-valued.) Strictly speaking you need to keep track of the separate cases, so that everything is uniquely defined, and then it all should come right:

If $x > 0$ then $u = x^2$ implies $\sqrt{u} = x$, $dx = du/(2*\sqrt{u})$ and we end up with $\int x^2 dx = 1/3*u^{(3/2)}$. Since what we know is that $\sqrt{u} = x$ (not just that $u = x^2$), when re-substituting we should write $1/3*u^{(3/2)}$ in terms of \sqrt{u} , giving $1/3*u^{(3/2)} = 1/3*\sqrt{u}^3 = 1/3*x^3$.

If $x < 0$ then $u = x^2$ implies $\sqrt{u} = -x$, $dx = -du/(2*\sqrt{u})$, and we end up with $\int x^2 dx = -1/3*u^{(3/2)}$. Now we have $-1/3*u^{(3/2)} =$

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$$-1/3*\sqrt{u}^3 = -1/3*(-x)^3 = +1/3*x^3, \text{ as before.}$$