Re: The Law of the Excluded Middle again (long)

Source: http://sci.tech−archive.net/Archive/sci.math/2007−12/msg00602.html

- *From*: Angus Rodgers <twirlip@xxxxxxxxxxx>
- *Date*: Mon, 03 Dec 2007 13:07:17 +0000

On Mon, 03 Dec 2007 04:24:27 −0500, quasi
<quasi@xxxxxxxx> wrote:

> On Sun, 02 Dec 2007 23:17:36 +0000, Angus Rodgers
> <twirlip@xxxxxxxxxxx> wrote:
>
>> On Sun, 02 Dec 2007 16:44:23 +0000, I wrote:
>>
>>> This application of the Law of the Excluded Middle [...]
>>> has a characteristic which I didn't notice at the time,
>>> and which no−one else has pointed out either: it is applied
>>> only
>>> to propositions containing only free variables.
>>
>> Inside the scope of quantification, and applied only to propositions
>> not themselves containing any quantifiers, it seems impossible (i.e.
>> even more impossible!) to equate (or confuse) truth with provability.
>
> But you managed to do it −− you are confusing it.

Perhaps, but not in the way you think (below).

>> There is no question of "proving" that x >= 1, when x is a free
>> variable not yet subject to quantification.

This is what I said, and you seem to be saying exactly the same
thing, perhaps under the impression that I meant something other
than what I said. (Probably this was because, as so often with
me, it just wasn't clear. But my struggles to say things clearly
only seem to result in lengthy verbiage, so I often have to say
things briefly, wait for misunderstandings to occur, and then

correct them − or else breathe a sigh of relief, if, for once,
I have managed to be both clear and brief.)

> If the variable x is free, there's no such thing as a valid statement of the form "x >= 1" unless the statement has an implied quantification. By default, the implied quantification is "for all", so the statement x >= 1 is false (and provably so) if the domain for the variable x is, for example, the set of real numbers.

I'm well aware of all of that.

> Quantification of free variables _must_ be specified, either explicitly or implicitly, otherwise you don't even have a valid statement.
>
> The distinction between truth and provability, as Goedel's results clearly show, is a real distinction, but applies only to _valid_ statements. Thus, lack of quantification doesn't yield an example of a non−provable statement, but rather an invalid statement.

All true, I'm sure, but irrelevant to my point.

I don't know how to explain my point better. Rather than try to do so (which would only lead to further possibly unreadable verbiage), may I simply ask how /you/ think of the meaning of (for example) the statement "either x > 1 or x <= 1", where x is a variable, which has been introduced in an informal proof, and you are still in the middle of the proof? No−one is asking for this statement to be frozen, quantified, and then assigned a truth value! That would be ridiculous: which seems to be what you are saying to me, but is something I already know (as does everybody else). But in the course of the proof, each of the statements "x > 1" and "x <= 1" is thought of as having a truth value (which in a sense is a "variable", just like x). At least, that is how I seem to think of things (so my unreliable faculty of introspection tells me). Do you think about it differently?

(It is of course quite possible that I am making some tremendously ridiculous howling error, but it is not the particular tremendously ridiculous howling error that you think it is!) :−)

(My whole aim in all of this is to locate precisely where I am thinking wrongly about mathematics; therefore, I am not offended in general terms by a suggestion that I have got something very badly wrong. But I am trying not to be prejudiced as to the location of my error(s), /and/ I am trying not to automatically believe someone when they say that some particular belief of

mine is erroneous. So I will argue about particular points − perhaps quite tenaciously − but always with the overall aim of eventually conceding defeat, and saying, "Ah, so THAT's it!")
−−
Angus Rodgers
(twirlip@ eats spam; reply to angusrod@)
Contains mild peril
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