Re: The Law of the Excluded Middle again (long)

Source: http://sci.tech−archive.net/Archive/sci.math/2007−12/msg00657.html

- *From*: quasi <quasi@xxxxxxxx>
- *Date*: Mon, 03 Dec 2007 15:18:17 −0500

On Mon, 03 Dec 2007 13:07:17 +0000, Angus Rodgers
<twirlip@xxxxxxxxxxx> wrote:

> On Mon, 03 Dec 2007 04:24:27 −0500, quasi
> <quasi@xxxxxxxx> wrote:
>
> > On Sun, 02 Dec 2007 23:17:36 +0000, Angus Rodgers
> > <twirlip@xxxxxxxxxxx> wrote:
> >
> > > On Sun, 02 Dec 2007 16:44:23 +0000, I wrote:
> > >
> > > > This application of the Law of the Excluded
> > > > Middle [...]
> > > > has a characteristic which I didn't notice at
> > > > the time,
> > > > and which no−one else has pointed out
> > > > either: it is applied only
> > > > to propositions containing only free
> > > > variables.
> > >
> > > Inside the scope of quantification, and applied only to
> > > propositions
> > > not themselves containing any quantifiers, it seems
> > > impossible (i.e.
> > > even more impossible!) to equate (or confuse) truth with
> > > provability.
> >
> > But you managed to do it −− you are confusing it.
>
> Perhaps, but not in the way you think (below).

No, let's look at your statement, _above_.

"Inside the scope of quantification, and applied only to propositions not themselves containing any quantifiers, it seems impossible (i.e. even more impossible!) to equate (or confuse) truth with provability."

To me, that's an almost meaningless statement, but to the extent that it has any meaning, it's false.

You talk about not being able to equate, for a statement of a certain form, truth with provability. But unless the statement is a legal statement of the given theory, there concept of either truth or provability. That's my objection in a nutshell.

> There is no question of "proving" that x >= 1, when x is a free variable not yet subject to quantification.

There's no such thing as a valid statement x >= 1 unless in the context of the proof, x has been chosen, possibly implicitly (hence is constant at that point), or else is quantified (and if the quantifier is omitted, the default quantification is assumed).

> This is what I said, and you seem to be saying exactly the same thing,

No, you said that in this context, it was impossible to equate truth with provability, and I said that was nonsense. Truth versus provability is a fascinating logical phenomenon that arises in certain theories (for example, first order incomplete theories), but not in all theories. In particular, your unquantified statement x >= 1 is _not_ an example of a statement where it's impossible to equate truth with provability.

> perhaps under the impression that I meant something other than what I said.

I find it difficult to decipher what you meant, so I simply objected to what you actually said:

" ... it seems impossible (i.e. even more impossible!) to equate (or confuse) truth with provability."

> ?(Probably this was because, as so often with me, it just wasn't clear.
> But my struggles to say things clearly only seem to result in lengthy
> verbiage, so I often have to say things briefly,

Briefly? When was that?

> wait for misunderstandings to occur, and then correct them −
> or else breathe a sigh of relief, if, for once, I have managed to be
> both clear and brief.)

The fact is, as I see it, you don't really want to be brief. You are
quite proud of your stream−of−consciousness ramblings.

If you really wanted to be brief, you would choose as models some
authors, professors, or sci.math posters, whose arguments are brief,
yet clear, and whose style you find appealing.

Had you truly been trying to emulate some of those models, your
arguments would have already begun to show some improvement, but as
far as I can see, they've actually been getting worse.

> > If the variable x is free, there's no such thing as a valid statement
> > of the form "x >= 1" unless the statement has an implied
> > quantification. By default, the implied quantification is "for all",
> > so the statement x >= 1 is false (and provably so) if the domain for
> > the variable x is, for example, the set of real numbers.

> I'm well aware of all of that.

But there's the confusion −− if a statement is invalid, you can't talk
about either truth or provability.

> > Quantification of free variables _must_ be specified, either
> > explicitly or implicitly, otherwise you don't even have a valid
> > statement.
> >
> > The distinction between truth and provability, as Goedel's results
> > clearly show, is a real distinction, but applies only to _valid_
> > statements. Thus, lack of quantification doesn't yield an example of a
> > non−provable statement, but rather an invalid statement.

> All true, I'm sure, but irrelevant to my point.

Who knows what your point is (well, I suppose you do), but as I said, I focused on what you actually said (truth vs provability). My point is that such a comparison is confusing, misleading and essentially wrong. It's not like you have uncovered some kind of trivial undecidability.

> I don't know how to explain my point better. Rather than try to do so (which would only lead to further possibly unreadable verbiage), may I simply ask how /you/ think of the meaning of (for example) the statement "either x > 1 or x <= 1", where x is a variable, which has been introduced in an informal proof, and you are still in the middle of the proof?

I would need to see a complete proof.

Here's your opportunity to keep it simple.

Let's see a very short and simple proof, with a minimum of clutter, that introduces the issue you describe above. You should be able to do it a few lines.

> No−one is asking for this statement to be frozen, quantified, and then assigned a truth value! That would be ridiculous: which seems to be what you are saying to me, but is something I already know (as does everybody else). But in the course of the proof, each of the statements "x > 1" and "x <= 1" is thought of as having a truth value (which in a sense is a "variable", just like x). At least, that is how I seem to think of things (so my unreliable faculty of introspection tells me). Do you think about it differently?

As I said, I would need to see a complete example.

> (It is of course quite possible that I am making some tremendously ridiculous howling error, but it is not the particular tremendously ridiculous howling error that you think it is!) :−)

Stop talking already −− where's the example?

> (My whole aim in all of this is to locate precisely where I am thinking wrongly about mathematics; therefore, I am not offended in general terms by a suggestion that I have got something very badly wrong. But I am trying not to be prejudiced as to the location of my error(s), /and/ I am trying not to automatically

> believe someone when they say that some particular belief of mine is erroneous. So I will argue about particular points − perhaps quite tenaciously − but always with the overall aim of eventually conceding defeat, and saying, "Ah, so THAT's it!")

Sorry, I don't believe you. In my opinion, you came to argue and indulge your narcissism.

But that's ok, have fun.

quasi
.