

Re: The Law of the Excluded Middle again (long)

Source: <http://sci.tech-archive.net/Archive/sci.math/2007-12/msg00711.html>

- *From:* Angus Rodgers <twirlip@xxxxxxxxxxx>
 - *Date:* Mon, 03 Dec 2007 23:32:05 +0000
-

On Mon, 3 Dec 2007 14:12:38 -0800 (PST), Randy Poe <poespam-trap@xxxxxxxxxxx> wrote:

On Dec 3, 8:07 am, Angus Rodgers <twir...@xxxxxxxxxxx> wrote:

On Mon, 03 Dec 2007 04:24:27 -0500, quasi

I don't know how to explain my point better. Rather than try to do so (which would only lead to further possibly unreadable verbiage), may I simply ask how /you/ think of the meaning of (for example) the statement "either $x > 1$ or $x \leq 1$ ", where x is a variable, which has been introduced in an informal proof, and you are still in the middle of the proof? No-one is asking for this statement to be frozen, quantified, and then assigned a truth value!

I can't understand what is bothering you about such a statement. I would say that of course it has a truth value. And if it is a valid proof, then that truth value better be "T".

Why do you think we can't say "either $x > 1$ or $x \leq 1$ " in a proof? If x is a real number, there aren't any other possibilities.

We are apparently in heated agreement! Nothing at all bothers me about such a statement occurring in a proof! But apparently, according to constructivists such as Keith Ramsay and Galathea, it /ought/ to bother me; and, as both those people are better mathematicians than I am, it bothers me that they think that it should bother me, when it doesn't.

A refined logical sensitivity (such as that of either of those two posters, or that of Brouwer, Heyting et al.) which rejects

Re: The Law of the Excluded Middle again (long)

the Law of the Excluded Middle as invalid would be the "dog that didn't bark in the night", which I keep going on about. I can't understand what I am supposed to see as being objectionable in the LEM. But the effort to understand that mystery obliges me to explain my own beliefs – in a way which I find educational, even if quasi (who is presumably not alone in his opinion, and generally seems a reasonable person, except now in relation to me) believes it to be merely perversely self-gratifying.

I'm truly sorry that I don't know how to write about things that interest me without using /lots/ of words! My style has been the subject of some complaints in other newsgroups as well as sci.math (although usually in more friendly and less insulting terms than those which quasi has just used). I am not just doing it to annoy! And I am sorry that my meaning has not been plain to you (and again, I doubt if you are alone in that).

As far as I understand it (which is not far), the reason why I /should/ be bothered by such a statement (although I am not!) is that you have to consider it, in its context, as applying to what Keith Ramsay called "a hypothetical [...] situation in which a real number x has been given. That is to say, a certain kind of construction has been made".

I think I understand the "hypothetical situation" part (which is more or less how I would have expressed the matter myself).

Obviously it is the "construction" part that is the issue.

I still just tend to visualise ordered triples of real numbers (x, y, z) (such as occur in the sample proof under discussion) as somehow objectively ranging over a region of mathematical "space", which exists objectively. (Here I am setting aside some worries about set theory and the definition of ordered triples ...)

But apparently I should instead think of a hypothetical situation in which someone (not necessarily a single individual, but some kind of generalised subject) has actually constructed some numbers x , y and z . In such a situation, it is not possible to say with certainty either that $x > 1$ or that $x \leq 1$, because either of these statements would require a proof, which simply might not be available (to the person or "subject" in question).

(Take all this with a pinch of salt, of course, because I am essentially only repeating something I have been told but have not yet understood.)

From this perspective, I think I can understand why my argument

Re: The Law of the Excluded Middle again (long)

about free variables and so on was a red herring (because in the "hypothetical situation", x is no longer a "free variable", but denotes an actually constructed real number). However, I still have a long way to go in understanding the perspective.

I still just don't get why I should always think of real numbers, in particular, as having actually been constructed by someone (in an actual or hypothetical situation).

And why did you specify "informal"?

Only because the specific proof I was using as an example (from the thread " $x^y + y^z + z^x > 1$ ") was informal. Secondly also because I know embarrassingly little about formal logic. (I did learn some once, and made serious use of the Compactness Theorem for some now-forgotten purpose, but I've now also forgotten most of the little formal logic I once knew.) I'm trying to stick to what I know, as a base from which to explore what I don't know.

(I think it may also be worth emphasising informality because it draws attention to the thought processes of a human mathematician doing the human activity of mathematics, and reduces a possible temptation to think of a mathematical argument as some kind of external object, perhaps consisting of marks on paper. But this may be just another expression of my emotional aversion to formal logic, or at least to an overemphasis on formal logic.)

--

Angus Rodgers
(twirlip@ eats spam; reply to angusrod@)
Contains mild peril

.