zeta function paradox at s=−1

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The value of the zeta function at s=−1, 1+2+3+4+...., = −1/12.

Here is a paradox: what is the value of the infinite series
1+2+4+5+7+8+10+11+.... ?

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Answer #1:

Begin with 1+2+3+4+.... = −1/12.

Multiply both sides by 3:
3*(1+2+3+4+....) = 3*(−1/12)
3+6+9+12+.... = −1/4.

Subtract that last infinite series from the one we began with:
(1+2+3+4+....) − (3+6+9+12+....) = −1/12 − −1/4

And we have our answer:
1+2+4+5+7+8+10+11+.... = 1/6.

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Answer #2:

Begin with 1+2+3+4+.... = −1/12.

Multiply both sides by 2:
2*(1+2+3+4+....) = 2*(−1/12)
2+4+6+8+.... = −1/6.

Subtract the last infinite series from the one we began with:
(1+2+3+4+....) − (2+4+6+8+....) = −1/12 − −1/6
1+3+5+7+.... = 1/12.
We can express this series as Sum(n=1 to infinity) 2n−1.

Now take the series 1+2+4+5+7+8+10+11+....
We can express this series as Sum(n=1 to infinity) (3n−2)+(3n−1).
(3n−2)+(3n−1) = 6n−3 = 3(2n−1).

Therefore this series equals 3 times the above−mentioned series,
Sum(n=1 to infinity) 2n−1.

Thus we have our answer:
1+2+4+5+7+8+10+11+.... = 3*(1/12) = 1/4.

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So what is the value of 1+2+4+5+7+8+10+11+.... ?
Is it 1/6 or 1/4 ?

Jeff Caveney
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