

Re: Area of an envelope of a curve

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- *From:* Martin <sleziak@xxxxxxxxxxxxxxxx>
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On Dec 5, 10:08 pm, Martin <slez...@xxxxxxxxxxxxxxxx> wrote:

My question is, whether the problem I formulate bellow is well-known, has a name or where can I read more about it.

TIA

Martin

Let C be a smooth curve in a plane.

If I move a line segment of length $2d$ along this curve in a such way that

- * the center of the line segment is always on the curve
- * the line segment is perpendicular to the curve

What can be said about area of the curve?

My guess (what the intuition suggests);

If the radius of curvature at each point is at least d , then the area is

$$A = l \cdot 2d$$

where l denotes the length of the curve.

If this condition is not fulfilled, then the inequality

$$A \leq l \cdot 2d$$

holds.

If I want to formulate similar problem also for piecewise continuous curves (i.e. I do not have curvature in each point) one possibility would be to take the points such that distance from the curve is at most d . (I am in the similar situation as above, but I have added two half-circles.)

My guess is that again a similar inequality should be valid.

Was I right in my conjectures? Is this well-known or is it equivalent to some known result? How is it called?

To avoid some misunderstanding, let me include two simple exmples.

Keep in mind that we are always working in a plane.

I am afraid I have forgotten to emphasize that the line segment has a constant length.

Re: Area of an envelope of a curve

If C is a line segment as well, the resulting figure will be a rectangle.

If C is a circle and the length of the line segment is less than the diameter, then the resulting figure is an annulus.