Re: Area of an envelope of a curve

Source: http://sci.tech−archive.net/Archive/sci.math/2007−12/msg01117.html

---

- *From*: quasi <quasi@xxxxxxxx>
- *Date*: Wed, 05 Dec 2007 17:23:41 −0500

On Wed, 5 Dec 2007 14:06:17 −0800 (PST), Martin
<sleziak@xxxxxxxxxxxxx> wrote:

> On Dec 5, 10:08 pm, Martin <slez...@xxxxxxxxxxxxx> wrote:
>
>> My question is, whether the problem I formulate bellow is well−known,
>> has a name or where can I read more about it.
>> TIA
>> Martin
>>
>> Let C be a smooth curve in a plane.
>> If I move a line segment of length 2d along this curve in a such way
>> that
>> * the center of the line segment is always on the curve
>> * the line segment is perpendicular to the curve
>> What can be said about area of the curve?
>>
>> My guess (what the intuition suggests);
>> If the radius of curvature at each point is at least d, then the area
>> is
>> A = l*2d
>> where l denotes the length of the curve.
>> If this condition is not fulfilled, then the inequality
>> A <= l*2d
>> holds.
>>
>> If I want to formulate similar problem also for piecewise continuous
>> curves (i.e. I do not have curvature in each point) one possibility
>> would be to take the points such that distance from the curve is at
>> most d. (I am in the similar situation as above, but I have added two
>> half−circles.)
>> My guess is that again a similar inequality should be valid.
>>
>> Was I right in my conjectures? Is this well−known or is it equivalent
>> to some known result? How is it called?
>
> To avoid some misunderstanding, let me include two simple exmples.

> Keep in mind that we are always working in a plane.
> I am afraid I have forgotten to emphasize that the line segment has a constant length.
>
> If C is a line segment as well, the resulting figure will be a rectangle.
>
> If C is a circle and the length of the line segment is less than the diameter, then the resulting fingure is anulus.

I think your claimed result follows by partitioning your curve and summing the areas swept out for each segment of the partition. In each segment the curve can be approximated by a line segment, so the swept out area is approximately a rectangle with area consistent with the desired result. You would need to prove the sum of the overlapping areas approaches zero as the width of the partition approaches zero. For that, you need a smoothness condition, to insure that, as the width of the partition approaches zero, the successive line segments used in approximating the curve join at an angle approaching a straight angle.

Essentially, it's an exercise in calculating areas directly, as a limit of a Riemann sum.

Alternatively, there may be a theorem that instantly proves it.

quasi
.