# Re: Multiple infinities − one more look

***Source:*** http://sci.tech−archive.net/Archive/sci.math/2007−12/msg01657.html

- *From*: "Mike Terry" <news.dead.person.stones@xxxxxxxxxxxxxxxxxxx>
- *Date*: Sat, 8 Dec 2007 19:30:42 −0000

"Venkat Reddy" <vreddyp@xxxxxxxxx> wrote in message
news:31f69511−410b−4245−9b4c−1291716a041d@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

> On Dec 8, 8:33 am, "Mike Terry"
> <news.dead.person.sto...@xxxxxxxxxxxxxxxxxxx> wrote:
>
>> "Venkat Reddy" <vred...@xxxxxxxxx> wrote in message

news:052a8b26−92db−46a9−8caf−1ba9c7300927@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

>>> I see. I think order is related to how you choose to represent
>>> the
>>> number. If the real number is represented by a digit sequence
>>> of
>>> infinite length, we immediately have an order for all reals,
>>> since
>>> digit position have an order. For example, for the length of 6
>>> digits,
>>> one can generate these sequences by writing all possible
>>> permutations
>>> and combinations of digit sequences in an orderly fashion.
>>> This can be
>>> continued for lager length of digit sequences without limit.
>>> All
>>> possible digit sequences are guaranteed to appear somewhere
>>> in the
>>> list.
>>
>> No. This process only generates finite digit sequences. Real numbers

have

infinite digit sequences...

> Instead, if you choose to represent a real number by an algebraic
> equation, here also we have an order in representing the equation
> itself, so the resulting reals have an order. Likewise for
> transcendental as well − once you have a way of identifying the
> number, we have a way of ordering them.

It seems you are thinking of something like "computable numbers", and these are indeed countable. Real numbers need not be computable in this sense (i.e. having a finite program to output their digits in sequence).

> You may say you want to represent the real number as a point on the
> line such as the interval [0,1]. Here also we have an order. Cut the
> line into half and mark the 0.5 as your first real number. Cut the two
> pieces in to two equal parts, mark 0.25 and 0.75 as new real numbers.
> Continue the process for ever and you have an ordering of all points.

No. Real numbers need not be one of these points you construct, although your process will construct real numbers arbitrarily close to any given real number.

Let's identify a real number as a unique label for a cut in a line segment (spatial continuum) at a random location. The fact is, such cuts or splits or points exist only after you imagine them. The

> continuum doesn't already have them ready for us to count. This is similar to fact that we also do not have "ones" ready to count as natural numbers, but only after you imagine some discrete items.
>
> The natural ordering of such cuts, is exactly similar and opposite to ordering of natural numbers. Natural number sequence keeps introducing a new "one" in an attempt to fill the "empty continuum". A reciprocal process for this would be to introduce a new "zero" or split in an attempt to empty the "full continuum". However, introducing the splits wouldn't be by adding one at a time, but 2^n of those at a time. This follows from the observation that in natural number sequence we are adding a "one" to every "empty continuum", and hence we need to add a split to every "full continuum", that is, the pieces resulting from the previous cuts.

I'm sort of with you so far...

> This ensures that we can always imagine 2^n number of splits for every imaginable natural number n.

Yes...

> When the natural number hits infinity,
> then we have 2^oo as the number of points, all are perfectly ordered or sequenced.

This doesn't make sense. Natural numbers go on indefinitely − they do not "hit infinity" at any point, although we can look at the totality of all cuts (points) you've constructed in all (finite) generations of the splitting process. This will be a countable set of points, as you've realised, and it will correspond to the real numbers of the form (m/2^n) where m,n are natural numbers.

This is not all real numbers − e.g. 1/3 is not one of these points!

Think about this carefully − your cutting process can go on forever if you imagine it so, but it will *never* produce a cut at the number 1/3. So what have you shown by this? You have shown that there is an incomplete subset of R which is countable − well done! :−) Your incomplete subset is also "dense" in the real numbers, but there are other obvious examples of countable, dense, subsets of R (e.g. the rationals) so your example is not remarkable.

To take your bisection idea a bit further, one approach could be to use the cuts as follows:

− imagine you have a point in the continuum [0,1]

− you do your 1st generation bisection (i.e. at the point 1/2) and ask "is my point in the left or right portion of the bisection?" You record which it is (say L or R)

− then in the next generation you concentrate on the previous L or R chunk as appropriate, and that chunk will again be bisected (i.e. at 1/4 or 3/4), and you can ask "is my point in the new L or R portion?" Again you record which it is (L or R)

− this process can proceed indefinitely, and you will record an L or R for each cutting generation.

*NOTE* : it is possible for your point to always be in the interior of the cut segments for *every* generation of cutting. E.g. the number 1/3 will have this property.

While it is possible you will never cut exactly on your point, it is none the less true that the intervals produced at each generation of cutting get smaller and smaller, and if you record the sequence of Ls and Rs for your number, the *infinite* sequence of Ls and Rs will uniquely characterise your original number.

E.g. 1/3 would result in the sequence (L,R,L,R,L,R,L,R,L,R,....)

The number of such sequences is NOT countable of course.

So to sumarise:

1) the set of points corresponding to your cuts is countable but misses out most numbers in the continuum

2) the missing numbers can be identified with sequences of Ls and Rs, which you might think of as "paths" through your cuts which home in on (and ultimately uniquely identify) the real numbers. However, while the number of exact cut points is countable, the number of "paths" through the cuts is uncountable. This is the point you are missing.

Regards,
Mike.

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